Title: Uncertainty

AIMA: Chapter 14 (Section 14.1-14.3)

Introduction to Artificial Intelligence

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Dealing with the real world requires modeling uncertainty
$\longrightarrow$ UAI

FOL does not directly model uncertainty: facts are true or false

- Extensions: modal logic, fuzzy logic

- Decision theory: $\begin{cases} \text{Probability theory} \\ + \\ \text{utility theory} \end{cases}$

  Probability theory: mathematical basis for systems
  Utility theory: model desirability of goals (rewards, penalty) and likelihood of achieving them

Manifestations of uncertainty in real-world

- Insufficient knowledge: choice A? choice B?
- Incompleteness:
  too many conditions to be enumerated, some are unknown
- Incorrectness: dynamic world

Example: catch a flight from Omaha, go early vs. go earlier+wait

Do the right thing $\rightarrow$ rational agent
Rational decision depends on relative importance of

- various goals
- likelihood of achieving them (and to what degree)

Origins of uncertain knowledge

- Laziness: difficulty to enumerate all conditions
- Theoretical ignorance: unavailability of a complete theory
- Practical ignorance:
  to what extent a specific situation 'fits' in the theory?

Example: Diagnosis (in judgemental domains)
medical, law, automobile/jet-engine repair, etc.

Example: Dental diagnosis

- $\forall\, p\, Symptom(p, Touthache) \Rightarrow Disease(p, Cavity)$?
  Incorrect, not everyone has cavity
- $\forall\, p\, Symptom(p, Touthache) \Rightarrow Disease(p, Cavity) \lor Disease(p, GumDisease) \lor Disease(p, WisdomTooth) \lor etc..$?
  Problem: Unlimited list of possible causes
- Causal rule:
  $\forall\, p\, Disease(p, Cavity) \Rightarrow Symptom(p, Touthache)$?
  Wrong: Not all cavities cause pain..
  List all possible reasons why a cavity may or may not hurt..

Origins of uncertain knowledge

- Laziness: too many antecedents or consequences to list
- Theoretical ignorance: Medical science has no complete theory
- Practical ignorance: patient has not undergone all necessary tests

Dealing with uncertain knowledge

Modeling: Degrees of belief are associated to sentences
Tool: probability theory

Example: 80% chance $\exists$ cavity if patient has toothache

- 80% include cases $\begin{cases} \text{cavity causes toothache} \\ \text{cavity and toothache exist but unrelated} \end{cases}$
- 20% include all other causes of toothache

Belief derived from $\begin{cases} \text{statistical data} \\ \text{general rule} \\ \text{combination of evidence sources} \end{cases}$

Important distinction

Degree of belief is <u>different</u> from degree of truth:
The fact "Patient has cavity" is either true or false (binary)
but, our <u>belief</u> that he has is $\in [0 \ldots 1]$

Probability theory: facts are true or false (as in Logic),
degree of <u>belief</u> modeled as a value $\in [0 \ldots 1]$

Fuzzy Logic: models degree of <u>truth</u> as a value $\in [0 \ldots 1]$

Prior vs. posterior probability

Example: card playing, draw 1 card

Probability first card is Ace of Spade is 1/52

Open card: truth established

Probability second card is Ace of Spade depends on previous evidence

Prior probability (unconditional probability):

before evidence is obtained

Posterior probability (conditional probability):

after evidence is obtained

"Agent often has some evidence and needs to compute the conditional probability of the outcomes it cares about"

Decision making under uncertainty

Planning: finding a sequence of actions to achieve goal

Several sequences may exist that achieve the goal

- Certain world: any is acceptable
- Uncertain world: some more likely to achieve goal than others

In practice, FINAL Steps can be further distinguished according to their preconditions not specified in the goal, "desirable" outcome

Thus, Agent must specify preferences between different possible outcomes of various plans

Example: arriving in time vs. waiting time

Utility theory: every state is characterized by a degree of usefulness
Agent chooses Final step of maximal utility

Decision theory

Decision theory = probability theory + utility theory

Principle of Maximum Expected Utility

> An agent is rational if and only if it chooses the action that yields the highest expected utility, averaged over all the possible outcomes of the action.

DT-Agent

function DT-AGENT(*percept*) **returns** an *action*
  static: a set probabilistic beliefs about the state of the world

  calculate updated probabilities for current state based on
    available evidence including current percept and previous action
  calculate outcome probabilities for actions,
    given action descriptions and probabilities of current states
  select *action* with highest expected utility
    given probabilities of outcomes and utility information
  return *action*

Basic Notation

- Statements: sentences in propositional logic (here)
  Represented by a symbol $A = (HasLynnCavity)$

- Belief: $\begin{cases} \text{prior probability} P(A) \\ \text{conditional probability} P(A|B) \end{cases}$
  $P(A)$ is used when there is no other information
  $P(A|B)$ is used as soon as some new information B is known

Prior probability (unconditional probability)

Example: $A = (HasLynnCavity)$, $P(A) = 0.1$

Propositions can include equalities involving random variables

Random variable:

$Weather$, **domain** $\langle Sunny, Rain, Cloudy, Snow \rangle$ (discrete/cont.)

$P(Weather = Sunny)= 0.7$, $P(Weather = Rain)= 0.2$,

$P(Weather = Cloudy)= 0.08$, $P(Weather = Snow)= 0.02$

Probability distribution of a random variable:

All its possible values: $\mathbf{P}(Weather) = \langle 0.7, 0.2, 0.08, 0.02 \rangle$

Propositions themselves as random variables: domain $\langle True, False \rangle$

$P(Cavity) \equiv P(Cavity = True)$, $P(\neg Cavity) \equiv P(Cavity = False)$

Probability of complex sentences: $P(Cavity \wedge \neg Insured)$=0.06

Conditional Probability (Posterior probability)

As soon as we have some evidence about previously unknown facts in the domain, prior probabilities are no longer applicable
$\rightarrow$ use conditional probability

We know nothing about the domain: $P(A)$ $\qquad (P(A|))$
We **know** B: $P(A|B)$
Moreover, we know C: $P(A|B \wedge C)$...

Example: $\begin{cases} P(Cavity) = 0.1 \\ P(Cavity|Toothache) = 0.8 \\ P(Cavity|Toothache \wedge Cavity) =? \end{cases}$

$\mathbf{P}(X|Y)$ = 2-dim table of $P(X = x_i|Y = y_j)$

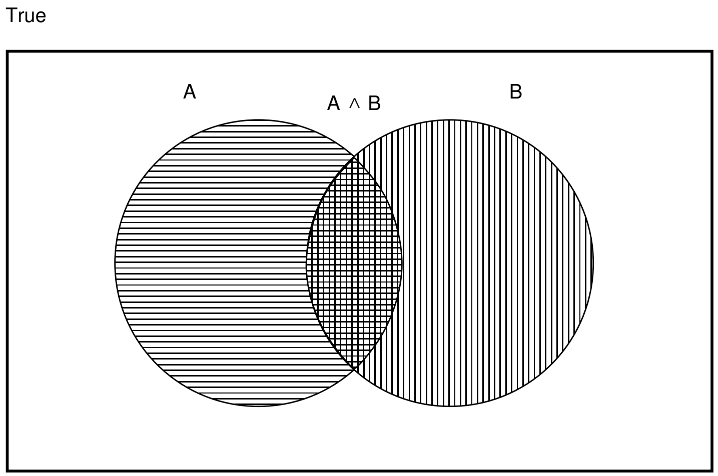
Product rule: $P(A \wedge B) = P(A|B).P(B) = P(B|A).P(A)$

Axioms of probability

1. $0 \leq P(A) \leq 1$

2. Valid proposition: $P(True) = 1$,
   unsatisfiable proposition $P(False) = 1$

3. $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$
   Venn diagram of possible words where A, B hold:



Properties of probability

From axioms, we can derive all other properties..

For example: $P(\neg A) = 1 - P(A)$?

$$\begin{aligned}
P(A \vee \neg A) &= P(A) + (P(\neg A) - P(A \wedge \neg A)) \\
P(True) &= P(A) + P(\neg A) - P(False) \\
1 &= P(A) + P(\neg A) \\
P(\neg A) &= 1 - P(A)
\end{aligned}$$

Joint probability distribution (Joint)

Specifies probability assignments to <u>all</u> propositions in the domain, simple and complex

- **Probabilistic model** of a domain:
  Set of random variables $\{X_1, \ldots, X_n\}$
- **Atomic event:** complete specification of the domain
  $X_i \leftarrow v_i$, for all X_i
- **Joint prob. dist.**: assigns prob. to all possible atomic events
  $\mathbf{P}(X_i) =$ 1-dim vector of all values of X_i
  Joint: n-dim table of the probability of events, each
  cell gives the probability of the event occurring

Example

	Toothache	¬Toothache
Cavity	0.04	0.06
¬Cavity	0.01	0.89

Joint: Properties

	Toothache	¬Toothache
Cavity	0.04	0.06
¬Cavity	0.01	0.89

Atomic events are:

- MUTEX $\Rightarrow$ conjunction of events necessarily false
- collectively exhaustive $\Rightarrow$ disjunction is true $\Rightarrow$ entries sum to 1

Probability of a statement?

Express statement as disjunction of atomic events, add up probabilities

$\rightarrow$ sum values in corresponding entry

$P(Cavity)=$ 0.06+0.04=0.1

$P(Cavity \vee Toothache) = ?$

$P(Cavity|Toothache) = \frac{P(.........)}{P(.........)} =....$

If we have the joint, we can compute probability of any statement

In real problem, many variables, joint has 2^n entries, too big to represent

Probabilistic reasoning systems work directly on conditional probabilities

Bayes rule (law, theorem)

A, B propositions:

$$P(A \wedge B) = P(A|B).P(B) = P(B|A).P(A)$$

X, Y random variables, we have the set of equations:

$$\mathbf{P}(X, Y) = \mathbf{P}(X|Y).\mathbf{P}(Y) = \mathbf{P}(Y|X).\mathbf{P}(X)$$

More generally, given a background evidence E:

$$\mathbf{P}(Y|X, E) = \frac{\mathbf{P}(X|Y, E).\mathbf{P}(Y|E)}{\mathbf{P}(X|E)}$$

Example: medical diagnosis

Causal rules:

Meningitis causes stiff neck 50% of the time, $P(S|M) = 0.5$

Probability of meningitis is $\frac{1}{50{,}0000}$, $P(M) = \frac{1}{50{,}0000}$

Probability of stiff neck is $\frac{1}{20}$, $P(S)= \frac{1}{20}$

Probability of meningitis given patient has stiff neck =

$$P(M|S) = \frac{P(S|M).P(M)}{P(S)} = 0.0002$$

$\rightarrow$ One out of 5000 patients with a stiff neck has meningitis

Note: Stiff neck is strongly indicated by meningitis, $P(S|M)$ high, but probability of meningitis in patient remains small, because $P(S)$ is much higher than $P(M)$

Epidemic of meningitis:

$P(M)$ goes up, but $P(S|M)$ remains the same

$P(S|M)$ **must be updated**

List of terms for glossary:

Due date: Monday, April 24, 2000

Words in margins in AIMA