Title: Uncertainty (II)
AIMA: Chapter 14

Introduction to Artificial Intelligence
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Bayes rule:

$$P(Cause|Effect) = \frac{P(Effect|Cause)P(Cause)}{P(Effect)}$$

Normalization

Suppose we want to computer a posterior distribution over $A = \langle a_1, a_2, \ldots, a_m \rangle$ given $B = b$

We can apply Bayes' rule for each value of A:

$$
\begin{aligned}
P(A = a_1 | B = b) &= \frac{P(B = b | A = a_1) P(A = a_1)}{P(B = b)} \\
&\vdots \\
P(A = a_m | B = b) &= \frac{P(B = b | A = a_m) P(A = a_m)}{P(B = b)}
\end{aligned}
$$

Adding these up, and given $\sum_i P(A = a_i | B = b) = 1$, we have:

$$\frac{1}{P(B=b)} = \frac{1}{\sum_i P(B=b|A=a_i) P(A=a_i)}$$

Normalization factor, α:

$$\mathbf{P}(A | B = b) = \alpha \, \mathbf{P}(B = b | A) P(A)$$

Typically, compute an unnormalized distribution, normalize at the end. Suppose we have:

$\mathbf{P}(B = b | A) \mathbf{P}(A) = \langle 0.4, 0.2, 0.2 \rangle$

Compute: $\mathbf{P}(A | B = b) = ?$

Bayes' rule for the multi-valued case:

$$\mathbf{P}(Y|X) = \alpha\ \mathbf{P}(X|Y)P(Y)$$

Diagnosis Y = Cause, X=Symptom

Using Bayes' rule for combining evidence:

$$\mathbf{P}(Z|X,Y) = \alpha\ \mathbf{P}(Z)\mathbf{P}(X|Z)\ \mathbf{P}P(Y|Z)$$

α is a normalization constant such that $\mathbf{P}(Z|X,Y)$ sum to 1.

Provided X and Y are conditionally independent given Z

$$\mathbf{P}(X|Y,Z) = \mathbf{P}(X|Z)$$
$$\mathbf{P}(Y|X,Z) = \mathbf{P}(Y|Z)$$

Diagnosis: Z = Cause, X, Y=symptoms

Product and Bayes rules

- Product rule: $P(A \wedge B) = P(A|B)P(B)$
- Condionalized product rule: $P(A \wedge B|E) = P(A|B \wedge E)P(B|E)$
- Bayes rule: $P(A|B) = \frac{P(B|A)P(B)}{P(A)}$
- Condionalized Bayes rule: $P(A|B, E) = \frac{P(B|A \wedge E)P(A|E)}{P(B|E)}$

Conditionalized versions:
derivations with respect to a background of evidence