Summary on local consistency

(Non-exhaustive)

Theory and Practice of Constraint Satisfaction
CSCE 990-05, Fall 1999
www.cse.unl.edu/~choueiry/CSE990-05/

General

- Node consistency
- Arc-consistency
  general: AC-1, AC-2, AC-3, AC-4, etc.
  Non-binary: GAC4 (Mohr & Masini'88)
- Path-consistency: PC-1, PC-2, PC-3

Specialized cases

- Arc-consistency:
  Monotone (Deville & van Hentenryck'91)
  Functional (Deville & van Hentenryck'91), (Liu'95)
  Anti-functional (van Hentenryck et al. '92)
  All-diff (Régin'94)
- Path consistent: (path-length=2 consistent) $\wedge$ complete net

Globally consistent network: (= decomposable, = strongly n-consistent, perfectly explicit) any partial consistent instantiation of a subset of variables can be extended to a solution with no backtracking.

Minimality, defined by Montanari'74 for binary networks, guarantees that each pair of values allowed by the constraints participates in at least one solution.

A decomposable network is always minimal.
The opposite is not true.

Islands of tractability

Question: Under which conditions global consistency and minimality are tractable?

1. Restrictions on the types of constraints
2. Restrictions on the constraint network topology
3. Restrictions on both

Sufficient conditions

Restrictions on constraint types

- Arc-consistency $\Rightarrow$ satisfiability:

  functional
monotonic } constraints (Deville & van Hentenryck'91)

- Path-consistency $\Rightarrow$ satisfiability: bijective constraints (David'93)

- Path-consistency $\Rightarrow$ global consistency:

  Monotone constraints (Montanari'74)

  Constraints that specify convex continuous intervals (Dechter et al.'90, van Beek'89)

  Row-convex constraints for discrete domains (van Beek'92)

Restrictions on constraint network topology

- Path-consistency on trees ⇒ minimality (Montanari'74)
- Relation(degree-of-local-consistency, width) to ensure global consistency (Freuder'82)

  Arc-consistency ∧ width=1 ⇒ backtrack-free (Freuder'82)

  (i,j)-consistency ∧ $i = j$-width ⇒ j-bounded backtrack (Freuder'85)

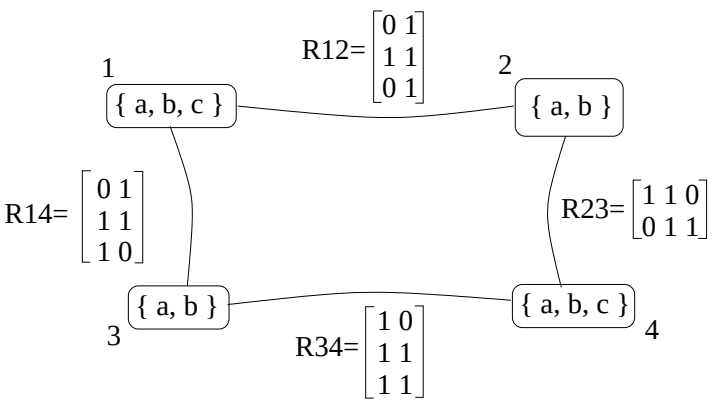
Restrictions on both (constraint type + network topology)

Solutions can be found backtrack-free for constraints networks that can be formulated as a causal theory (Dechter & Pearl'91)

DAG with any constraints
or
Special topology with special constraint types (causality)

Generally

Main work in this area: Dechter, Freuder, van Beek, Montanari, Sam-Haroud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R12= 0 1
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0 1
2
{ a, b }
R14= 0 1
1 1
1 0
R23= 1 1 0
0 1 1
{ a, b }
3
R34= 1 0
1 1
1 1
{ a, b, c }
4