# Summary on local consistency

(Non-exhaustive)

Berthe Y. Choueiry (Shu-we-ri)
Ferguson Hall, Room 104
choueiry@cse.unl.edu, Tel: (402)472-5444

## General

- Node consistency
- Arc-consistency
  general: AC-1, AC-2, AC-3, AC-4, etc.
  Non-binary: GAC4 (Mohr & Masini'88)
- Path-consistency: PC-1, PC-2, PC-3

## Specialized cases

- Arc-consistency:
  Monotone (Deville & van Hentenryck'91)
  Functional (Deville & van Hentenryck'91), (Liu'95)
  Anti-functional (van Hentenryck et al. '92)
  All-diff (Régin'94)
- Path consistent: (path-length=2 consistent) $\wedge$ complete net

**Globally consistent network:** (= decomposable, = strongly $n$-consistent, perfectly explicit) any partial consistent instantiation of a subset of variables can be extended to a solution with no backtracking.

**Minimality**, defined by Montanari'74 for binary networks, guarantees that each pair of values allowed by the constraints participates in at least one solution.

A decomposable network is always minimal.
The opposite is not true.

# Islands of tractability

**Question:** Under which conditions global consistency and minimality are tractable?

1. Restrictions on the types of constraints
2. Restrictions on the constraint network topology
3. Restrictions on both

Sufficient conditions

# Restrictions on constraint types

- Arc-consistency $\Rightarrow$ satisfiability:
  $\left.\begin{array}{l}\text{functional}\\ \text{monotonic}\end{array}\right\}$ constraints (Deville & van Hentenryck'91)
- Path-consistency $\Rightarrow$ satisfiability: bijective constraints (David'93)
- Path-consistency $\Rightarrow$ global consistency:
  Monotone constraints (Montanari'74)
  Constraints that specify convex continuous intervals (Dechter et al.'90, van Beek'89)
  Row-convex constraints for discrete domains (van Beek'92)

# Restrictions on constraint network topology

- Path-consistency on trees $\Rightarrow$ minimality (Montanari'74)
- Relation(degree-of-local-consistency, width) to ensure global consistency (Freuder'82)

Arc-consistency $\wedge$ width=1 $\Rightarrow$ backtrack-free (Freuder'82)

(i,j)-consistency $\wedge$ $i = j$-width $\Rightarrow$ $j$-bounded backtrack (Freuder'85)

## **Restrictions on both** (constraint type + network topology)

Solutions can be found backtrack-free for constraints networks that can be formulated as a causal theory (Dechter & Pearl'91)

DAG with any constraints
or
Special topology with special constraint types (causality)

## **Generally**

Main work in this area: Dechter, Freuder, van Beek, Montanari, Sam-Haroud