Title: Intelligent Backtracking Algorithms
CSP: finite domain, binary constraints

Material: Prosser CI'93, Tsang Chapter 5

Foundations of Constraint Satisfaction
CSCE 990-005, Fall 2000

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Search: two main mechanisms

1. Backtracking: when encountering dead-ends
2. Checking:
   - Back-checking: against past variables
   - Look-ahead: against future variables

$$\begin{cases} \text{Full: AC-3 on all future variables (MAC)} \\ \text{Partial: } \begin{cases} \text{DAC} \\ \text{Forward checking} \end{cases} \end{cases}$$

Improve performance of search

1. Backtracking: Naive/Chronological backtracking (BT), backjumping (BJ), conflict-directed backtracking (CBJ)
2. Checking: Backmarking (BM), forward checking (FC)
3. Hybrids: BMJ, BM-CBJ, FC-BJ, BM-CBJ

BT	BJ	CBJ
BM	BMJ	BM-CBJ
FC	FC-BJ	**<u>FC-CBJ</u>**

Evaluation:
- Empirical, Prosser, 450 instances of Zebra
- Theoretical, Kondrak & Van Beek

BT → BJ → CBJ: more informed backtracking

BT → BM → FC: tree expansion, forward move

	Go back		
Go forward	BT	BJ	CBJ
	BM	BMJ	BM-CBJ
	FC	FC-BJ	**FC-CBJ**

Notation: in Prosser's paper

- Variables: V_i, $i \in [1, n]$
- Domain: $D_i = \{v_{i1}, v_{i2}, \ldots, v_{M_i}\}$
- Constraint between V_i and V_j: $C_{i,j}$
- Constraint graph: G
- Arcs of G: $\text{Arc}(G)$
- Instantiation order (static or dynamic)
- Shallow: nodes closer root in search tree
- Deep: nodes closer to leaves/fringe in search tree
- Language primitives: list, push, pushnew, remove, set-difference, union, max-list

Data structures: in Prosser's paper

- Vector of values v: $v[i]$ is value assigned to i^{th} variable, V_i
- $v[i]$ also indicates variable V_i
- Pseudo-variable $v[0]$ (root of tree): backtracking to $v[0]$ indicates insolvability
- domain[i]: array of the domain of variable V_i, domain[0] $=$ nil
- current-domain[i], domain of V_i <u>during</u> search
  when backtracking occurs, current-domain[i] of future variables needs to be reinstantiated.
- Function check(i,j): is true if the values assigned to variable V_i and V_j are consistent

Generic search: bcssp

```
PROCEDURE bcssp (n, status):
BEGIN
consistent ← true;
status ← unknown;
i ← 1;
WHILE status = unknown
      DO BEGIN
      IF consistent
            THEN i ← label(i, consistent)
            ELSE i ← unlabel(i, consistent)
      IF i> n
      THEN status ← "solution"
      ELSE IF i = 0 THEN status ← "impossible"
      END
END
```

Prosser's notation for algorithm X:

- forward move: X-label
- backward move: X-unlabel
- Parameter of (un)label: current variable, Boolean
- Return of (un)label: new current variable

Label terminates with:

- Consistent = true and returns $i + 1$
- Consistent = false, current-domain[i] = nil, and returns i

Unlabel:

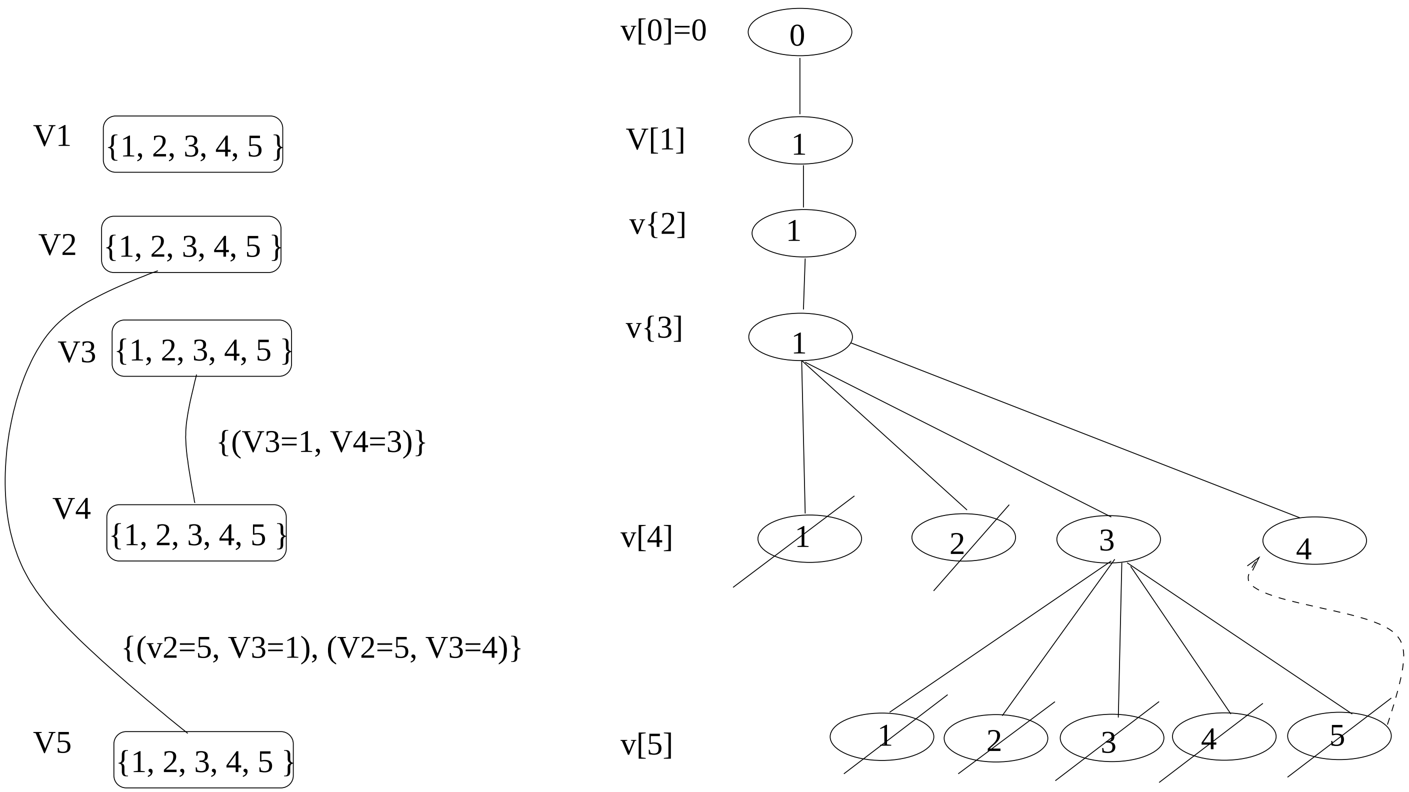
- is called when consistent = false and current-domain[i] = nil
- selects V_h to backtrack to
- Un-instantiates all variables $V_h < V_k \leq V_i$
- Removes $v[h]$ from current-domain[h]
- Sets consistent to true if current-domain[h] $\neq \emptyset$
- returns h

Chronological backtracking (BT): bt-label, bt-unlable

- When $v[i]$ is assigned a value x from current-domain[i] we perform back-checking of x against past variables (check(i,k))
- If back-checking succeeds, bt-label returns $i + 1$
- If back-checking fails, we remove x from current-domain[i], choose another value from current-domain[i], a repeat check
- If no other value exits, then current-domain[i] is refreshed (to domain[i]), $V_{h=i-1}$ is uninstantiated (assigned value is removed from current-domain[h]), and we seek an new values for V_h

All future variables j: current-domain[j] = domain[j]
All past variables g: current-domain[g] $\subseteq$ domain[g]

Example: BT


V1 {1, 2, 3, 4, 5 }
V2 {1, 2, 3, 4, 5 }
V3 {1, 2, 3, 4, 5 }
{(V3=1, V4=3)}
V4 {1, 2, 3, 4, 5 }
{(v2=5, V3=1), (V2=5, V3=4)}
V5 {1, 2, 3, 4, 5 }
v[0]=0
0
V[1]
1
v{2]
1
v{3]
1
v[4]
1 2 3 4
v[5]
1 2 3 4 5

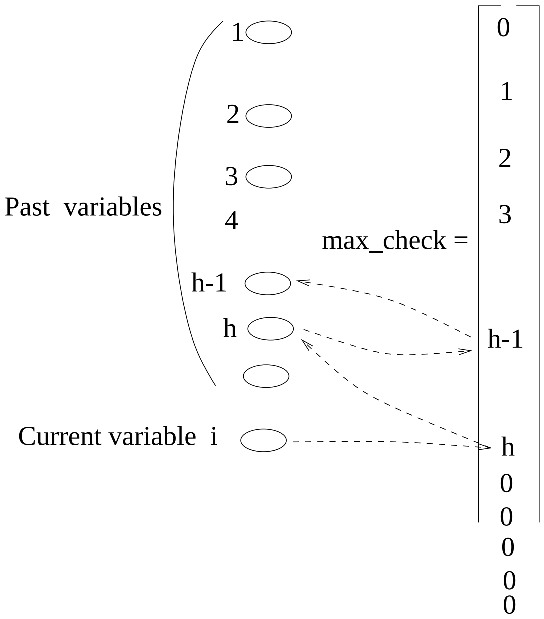
Note:
BT assumes that the instantiation of V_i was prevented by a bad choice for $h = (i-1)$. Thus it tries to change the assignment of V_h.

When this assumption proves wrong $\longrightarrow$ Thrashing

Backjumping (BJ) tries to avoid that:
$\longrightarrow$ jumps to the reason for failure then proceeds as BT

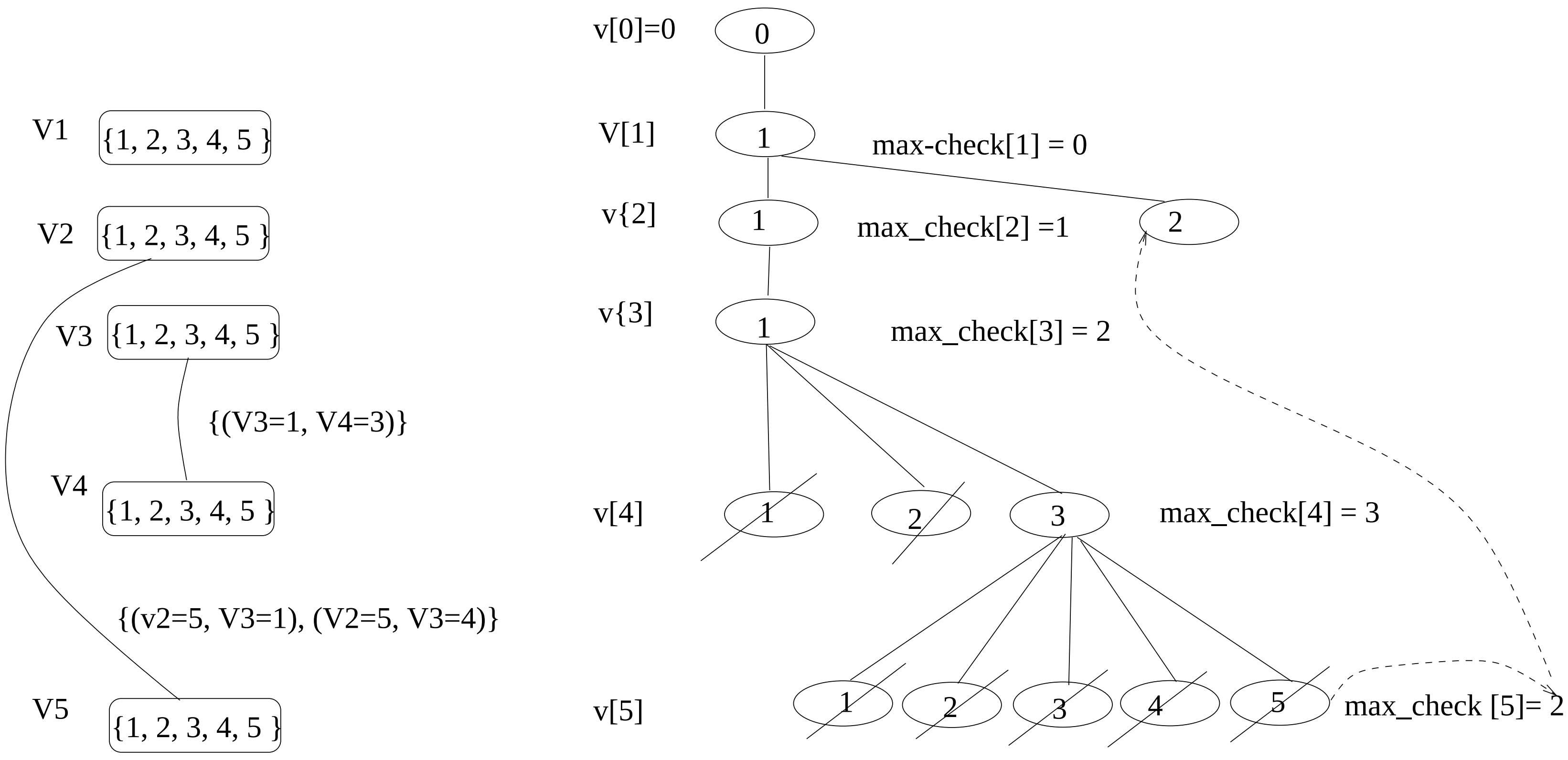
Backjumping (BJ) tries to reduce thrashing:

- When $v[i]$ is instantiated, BJ remembers V_h <u>deepest node</u> among past variables that $v[i]$ checked against.
- If current-domain[h] is getting empty, simple chronological backtracking from h is performed
- max-check[i]: global, initialized to 0
- at level i when check(i,h), max-check[i] $\leftarrow$ max(max-check[i], h)



$\longrightarrow$ **BJ jumps then steps**

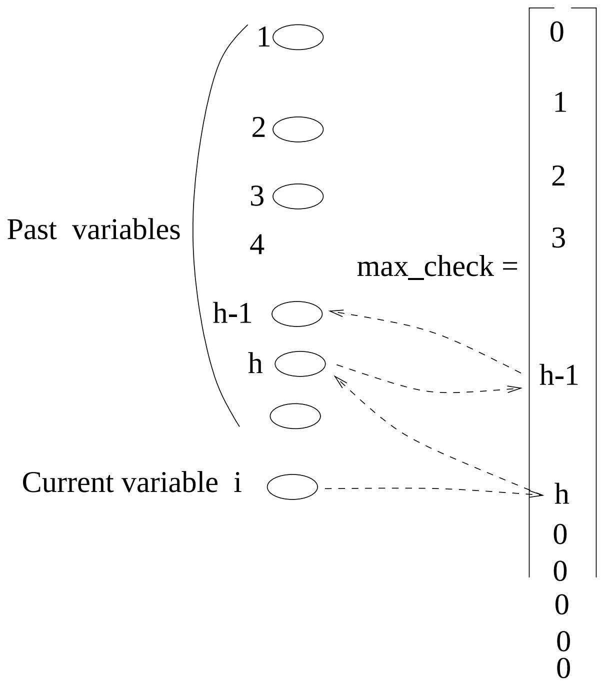
Example: BJ



V1
{1, 2, 3, 4, 5 }
V2
{1, 2, 3, 4, 5 }
V3
{1, 2, 3, 4, 5 }
{(V3=1, V4=3)}
V4
{1, 2, 3, 4, 5 }
{(v2=5, V3=1), (V2=5, V3=4)}
V5
{1, 2, 3, 4, 5 }
v[0]=0
0
V[1]
1
max-check[1] = 0
v{2]
1
max_check[2] =1
2
v{3]
1
max_check[3] = 2
v[4]
1
2
3
max_check[4] = 3
v[5]
1
2
3
4
5
max_check [5]= 2

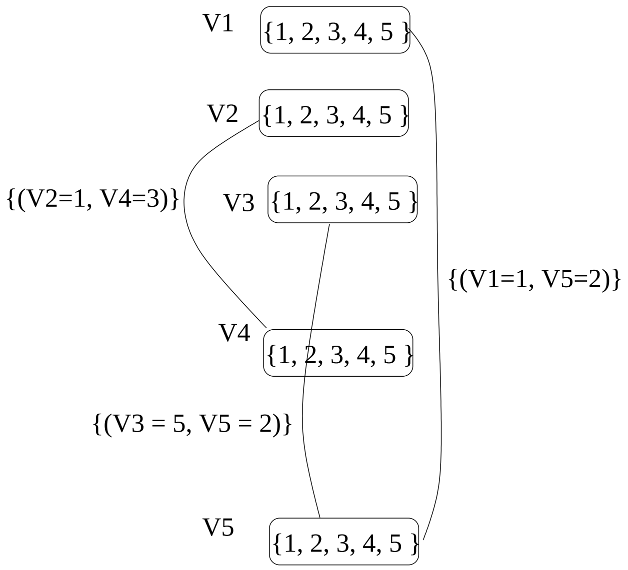
Backjumping (BJ)

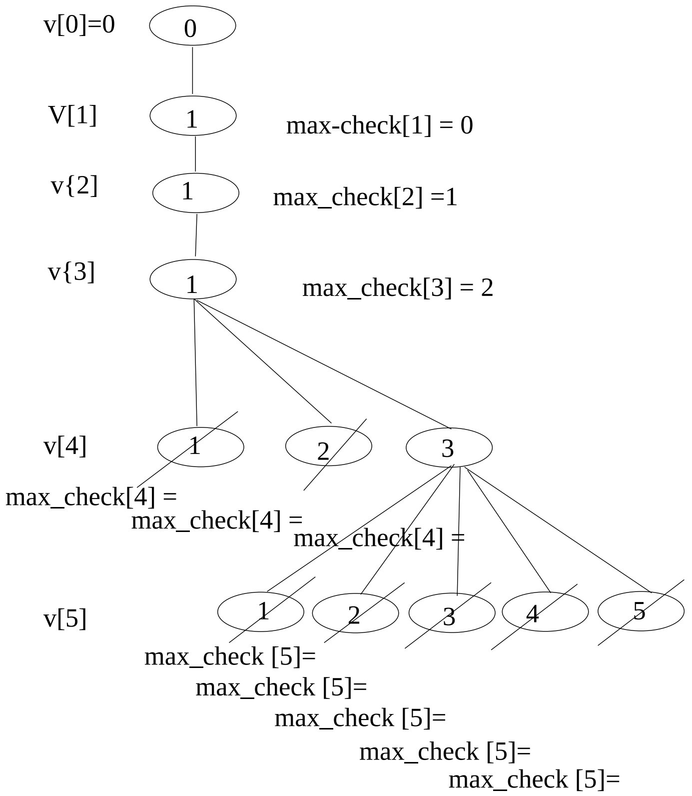
- bj-label, same as bt-label, but updates max-check[i]
- bj-unlabel, same as bt-unlabel, but
  — backtracks to h = max-check[i]
  — resets max-check[j] $\leftarrow 0$ for $j \in [h+1, i]$



Example: BJ (II)

D. Keshwani, 1999





Note: max-check[i] is:

– the deepest value i has **checked** against (max-check)

– not the deepest value i is in **conflict** with (max-fail)

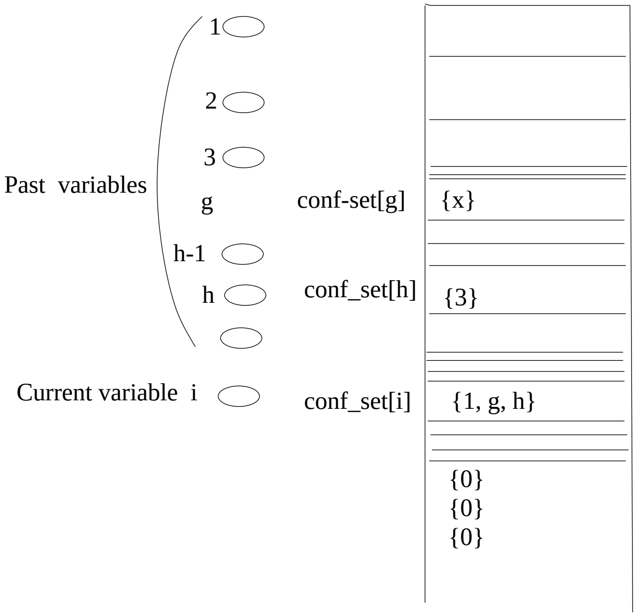
Conflict-directed backjumping (CBJ)

- BJ jumps from $v[i]$ to $v[h]$, then Steps from $v[h]$ to $v[h-1]$
- CBJ jumps from $v[i]$ to $v[h]$, and jumps again across **conflicts** involving **both** $v[i]$ and $v[h]$

CBJ

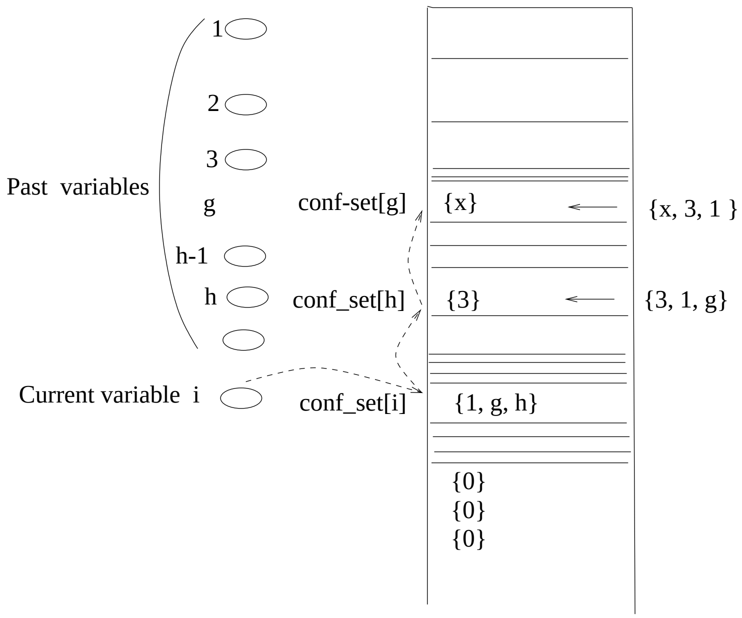
CBJ maintains conf-set[i] for each variable:

- conf-set first initialized to {0}
- conf-set[i] is subset of past variables in conflict with i



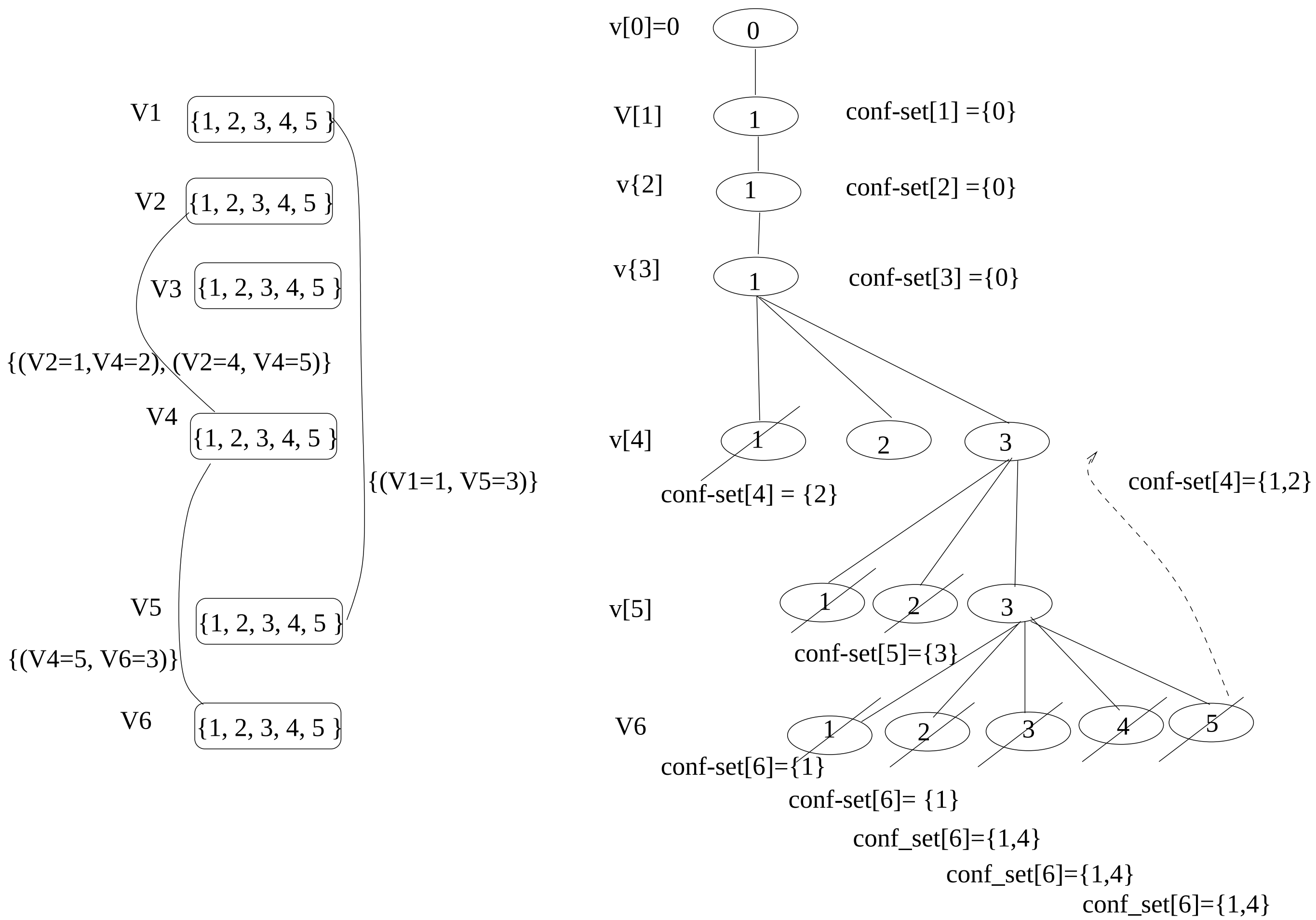
CBJ (cont.)

- when a check(i,h) fails, conf-set[i] $\leftarrow$ conf-set[i] $\cup\{h\}$
- when current-domain[i] $= \emptyset$, CBJ
  1. jumps to deepest past variable h in conf-set[i]
     $$h \leftarrow \text{max-list(conf-set[i])}$$
  2. updates conf-set[h] $\leftarrow$ conf-set[h] $\cup$ (conf-set[i] -h)
- as conf-set updated, primitive learning (while searching)



Example CBJ

D. Keshwani, 1999



Backtracking: summary

- Chronological backtracking:
  - steps to previous level
  - no extra data structures
- Backjumping:
  - jumps to deepest checked against variable, then steps back
  - uses array of integers: max-check[i]
- Conflict-directed backjumping:
  - jumps across deepest conflicting variables
  - uses array of sets: conf-set[i]

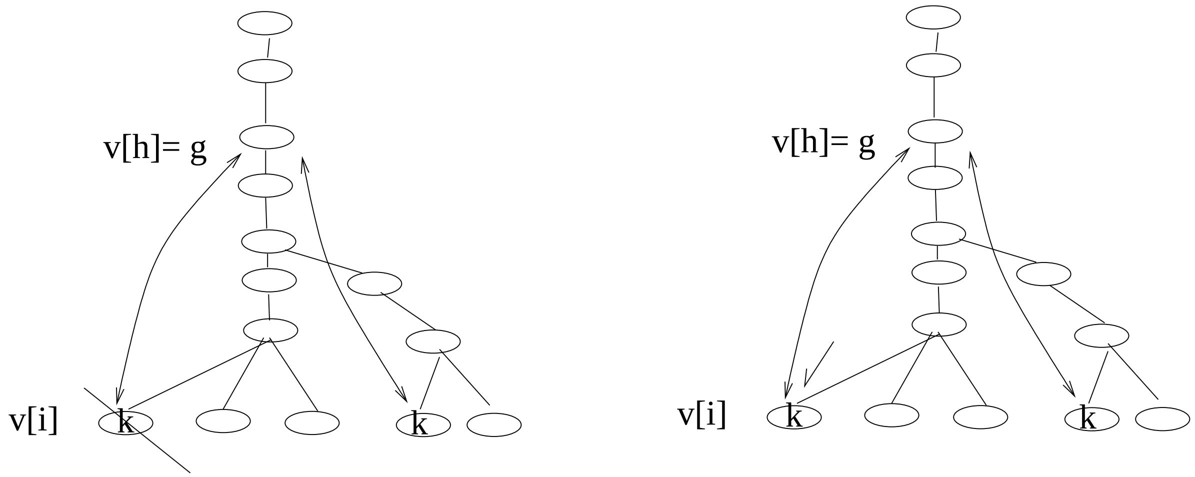
Backmarking tries to reduce amount of consistency checking

Consider:

Current variable V_i is about to be reinstantiated value k.

$V_i \leftarrow k$ was checked against $V_h \leftarrow g$

V_h has not been modified



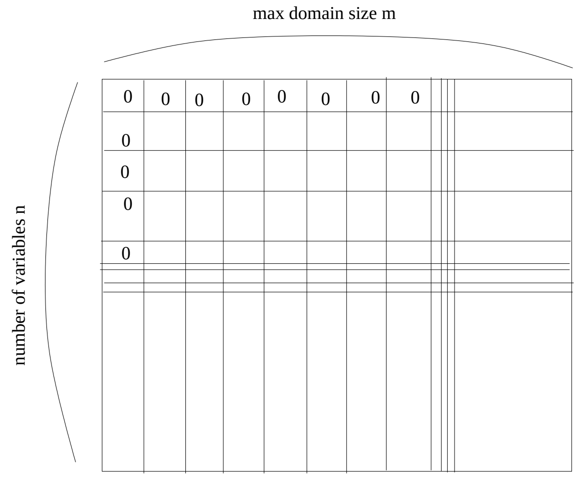
Two situations:

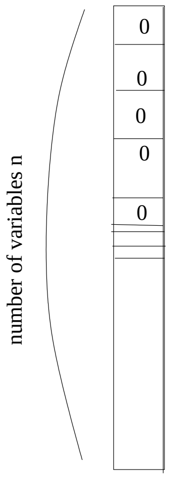
1. Either, $(V_i = k, V_h = g)$ has failed $\Rightarrow$ it will fail again, or
2. $(V_i = k, V_h = g)$ was consistent $\Rightarrow$ it will remain consistent

Backmarking: data structures, two arrays:

maximum checking level: *mcl* (dim. nxm)

minimum backup level: *mbl* (dim. n)

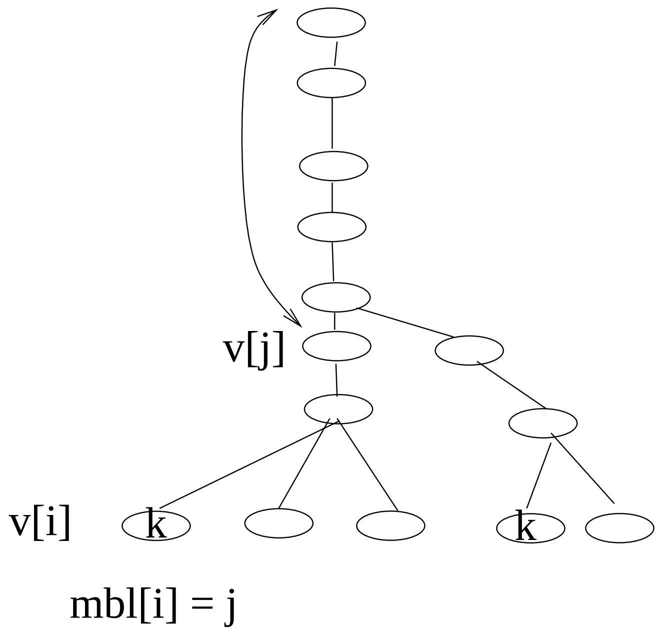




- mcl[i,k]: deepest variable v[i] ← k checked against
  → mcl[i,k] is a finer version of max-check[i]
- mbl[i]: shallowest past variable whose value has changed since v[i] was the current variable
- BM (and all its hybrids) do not allow dynamic variable ordering

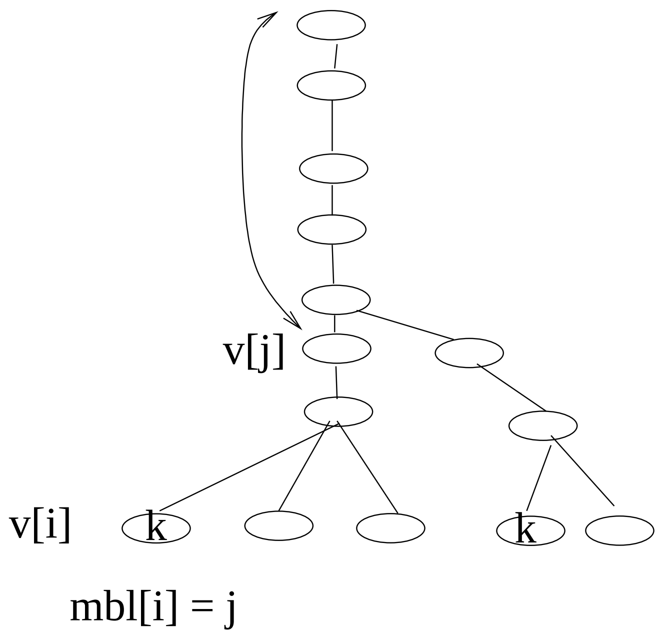
Maximum checking level: mcl[i,k] = j

BM is aware the deepest variable (v[i] $\leftarrow$ k) checked against is v[j]



Minimum backup level: mbl[i] = j

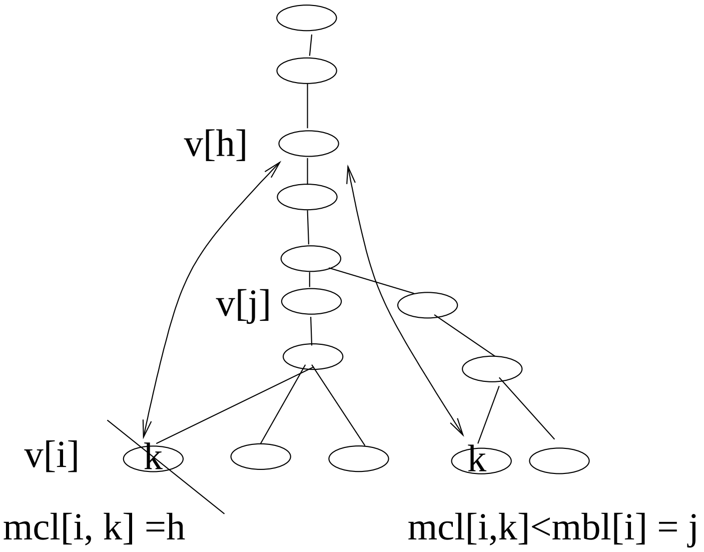
BM is aware that all variables in the past of v[j] have not changed values



Type a savings:

When mcl[i,k]$<$ mbl[i],

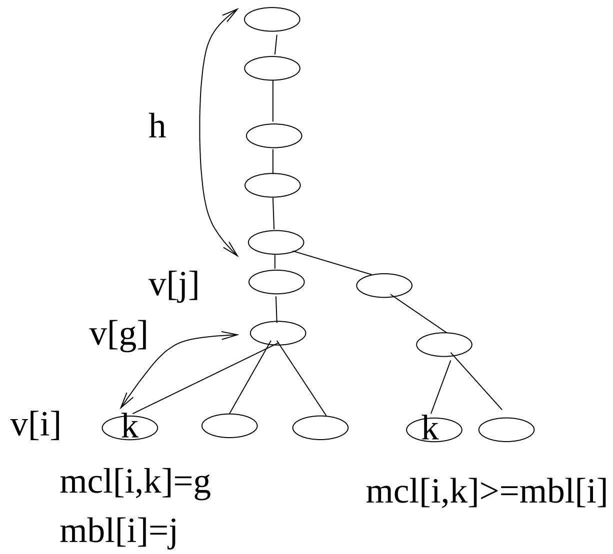
do not check v[i]$\leftarrow$ k, because it will fail



Type b savings:

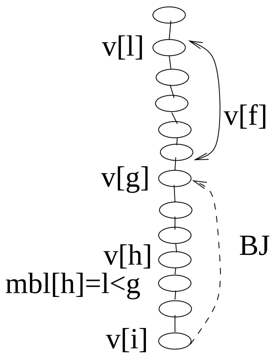
When mcl[i,k]$\geq$ mbl[i],

do not ckeck(i, h$<$j) because they'll succeed



Hybrids of BM

- mcl can be used to allow BJ within BM
- BJ and BM can be mixed, yielding BMJ: BMJ avoids redundant consistency checking (Type a + b savings) and reduces number of nodes visited during search (by jumping)
- Caveat: BMJ can perform worst than BM
  v[i] BJs to v[g], when v[h] is reconsidered, it will be checked against all f, v[l] $\leq$ f <g :–(



  BMJ enjoys <u>some</u> of the advantages of BM

- Phenomenon will worsen with CBJ

Forward checking (FC)

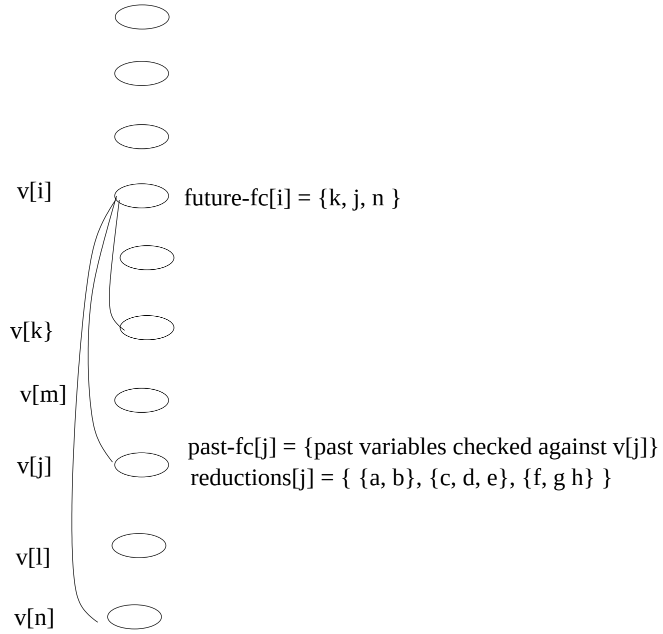
- Looking ahead: from current variable, consider all future variables and clear from their domains the values that are not consistent with current partial solution.
- FC makes more work at every instantiation, but will expand less nodes
- When FC moves forward, the values in current-domains of future variables are all compatible with past instantiations, thus saving back-checking
- FC may "wipe out" the domain of a future variable (a.k.a. domain annihilation) and thus discover conflicts early on. FC then backtracks chronologically
- Goal of FC is to fail early (avoid expanding fruitless subtrees)

Forward checking data-structures

When v[i] $\leftarrow$ k,

current-domain[j] is filtered $(i < j \leq n)$

- reduction $\in$ reductions[j] is a set of values removed from current-domain[j] by some variable before v[j]
- future-fc[i]: subset of future variables v[i] checks against
- past-fc[j]: past variables that checked against v[j]
- all these sets are treated as stacks



Forward checking: functions

- check-forward
- undo-reductions
- update-current-domain
- fc-label
- fc-unlable

Forward checking

- Function: check-forward(i,j) is called when v[i] is instantiated,
  it performs REVISE (j, i) (ref. AC)
  returns false if current-domain[j] is empty, true otherwise
- values removed from current-domain[j] are pushed into reductions[j]
- these values will be popped if we have to backtrack over v[i] (undo-reductions)
- Function update-current-domain:
  current-domain[i] ← domain[i] - reductions[i]
  (actually, we have to iterate over reductions=set of sets)
- fc-label: attempts to instantiate current-variable, then
  filters domains of all future variables (push into reductions)
  whenever current-domain of a future variable wiped out, v[i] uninstantiated, and domain filtering undone (pop reductions)

Hybrids for FC

FC suffers from thrashing, since it is based on BT

- FC-BJ: max-check is integrated in fc-bj-label, fc-bj-unlabel
- FC-BJ: enjoys advantages of FC and BJ.. but suffers malady of BJ (jump then step)
- FC-CBJ: (best algorithm so far)
  fc-cbj-label and fc-cbj-unlable

Consistency checking: summary

- Chronological backtracking:
  - steps to previous level
  - no extra data structures
  - simple back-checking
- Backmarking:
  - uses mcl, and mbl
  - two types of consistency-checking savings
- Forward checking:
  - works more at every instantiations, but expands fewer subtrees
  - reductions[i], future-fc[i], past-fc[i], current-domain[i]

Experiments Empirical evaluation on Zebra problem:

- representative of design/scheduling problems
- 25 variables, 122 constraints
- binary constraints
- permutation of variable ordering yields new search spaces
- variable ordering: different (band/induced) 'width' of graph

450 problems instances were generated

Each algorithm was applied to each instance

Analysis of experiments

Algorithms compared with respect to:

1. number of consistency checks performed: (on average)
   FC-CBJ $<$ FC-BJ $<$ BM-CBJ $<$ FC $<$ CBJ $<$ BMJ $<$ BM $<$ BJ $<$ BT
2. number of nodes visited: (on average)
   FC-CBJ $<$ FC-BJ $<$ FC$<$ BM-CBJ $<$ BMJ $=$ BJ $<$ BM $=$ BT
3. timing: (on average)
   FC-CBJ$<$ FC-BJ $<$ FC$<$ BM-CBJ $<$ CBJ$<$BMJ$<$ BJ$<$ BT$<$ BM

Conclusions

- How about other look-ahead techniques: MAC, etc.
- how about other backtracking algorithms, (Graph-based backjumping, GBJ)
- FC-CBJ is (apparently) the champion
- Kondrak and Van Beek IJCAI'95 did the theoretical study