P542. (8.3)

1

3. List the ordered pairs in the relations on {1. 2. 3}. Corresponding to these matrices. (where the rows and columns corresponding to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

(1,1), (1,3), (2,2), (3,1), (3,3)

b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(1,2) (2,2) (3,2)

c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(1,1), (1,2), (1,3),

(2,1), (2,3),

(3,1), (3,2), (3,3)

P543.

13. Let R be the relation represented by the matrix

$$M_R = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Find the matrix representing :

a) R^{-1}

sol: $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

b) $\overline{R}$

sol: $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

c) R^2.

sol: $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

P553 (8.4)

3. Let R be the relation $\{(a,b) \mid a \text{ divides } b\}$ on the of integers. What is the symmetric closure of R?

solution: $R \cup R^{-1}$

$= \{(a,b) \mid a \text{ divides } b \text{ or } b \text{ divides } a\}$

P527.

8.1.3: Let $S = \{1, 2, 3, 4\}$. Determine whether each relation R on S has the listed properties.

$R = \{(1,1), (1,2), (2,1), (2,2), (3,3), (4,4)\}$

~~Solution~~

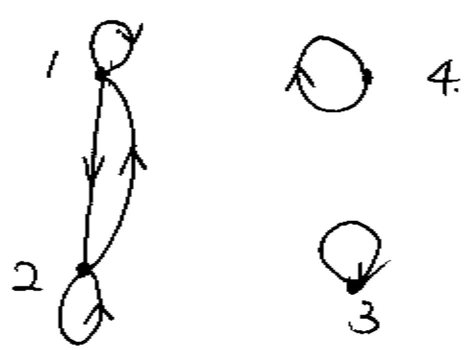
question 1: Draw the 0-1 matrix representation of R.

solution:

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

2: Draw the digraph representation of R.

solution:



3. Is R symmetric?

Yes

4. Is R transitive?

Yes.

5. Is R reflexive?

Yes

P529. 43.

a) How many relations are there on the set $\{a, b, c, d\}$

$$2^{n^2} = 2^{4^2} = 2^{16} = 65536$$

8.5

P563 9. Suppose that A is a nonempty set, and f is a function that has A as its domain. Let R be the relation on A consisting of all ordered pairs (x, y) such that $f(x) = f(y)$.

a) Show that R is an equivalence relation on A.

Solution: $(x, x) \in R$ because $f(x) = f(x)$.

Hence, R is reflexive.

$(x, y) \in R$ iff $f(x) = f(y)$, which holds iff $f(y) = f(x)$ iff $(y, x) \in R$.

Hence, R is symmetric.

If $(x, y) \in R$, and $(y, z) \in R$, then $f(x) = f(y)$ and $f(y) = f(z)$. Hence, $f(x) = f(z)$. Thus, $(x, z) \in R$.

It follows that R is transitive.

b) What are the equivalence classes of R?

the sets $f^{-1}(b)$ for b in the range of f.

P563

8.5:3 Which of these relations on the set of all functions from $\mathbb{Z}$ to $\mathbb{Z}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack

a) $\{(f,g) \mid f(1) = g(1)\}$

Solution → It is ~~reflexive~~. transitive.

$\because$ if we have $f\,R\,g$, i.e. $f(1) = g(1)$

and $g\,R\,h$, i.e. $g(1) = h(1)$

then we will have $f(1) = g(1) = h(1) \Rightarrow f(1) = h(1)$

then we have $f\,R\,h$.

→ It is Reflexive.

$\because$ ~~if~~ we have $f(1) = f(1)$, which implies $f\,R\,f$.

thus, it is reflexive.

→ It is symmetric:

$\because$ if we have $f\,R\,g$, i.e $f(1) = g(1)$

then we will have $g(1) = f(1)$, which implies $g\,R\,f$.

So. if $f\,R\,g$, ~~then~~ then $g\,R\,f$. $\therefore$ it is symmetric.

→ Since R ~~is~~ has all three properties, it is an equivalence relation.

(b) $\{(f,g) \mid f(0)=g(0) \text{ or } f(1)=g(1)\}$.

Solution: → It is reflexive:

$\because$ we have $f(0)=f(0)$ or $f(1)=f(1)$ then $f R f$.

so. it is reflexive.

→ It is symmetric.

$\because$ if we have $f R g$, ie. $f(0)=g(0)$ or $f(1)=g(1)$

then we have $g(0)=f(0)$ or $g(1)=f(1)$.

thus. $g R f$. So it is symmetric.

→ It is not transitive.

$\because$ if we have. $f R g$ ie. $f(0)=g(0)$ or $f(1)=g(1)$

and ~~we~~ $g R h$ ie. $g(0)=h(0)$ or $g(1)=h(1)$

Counter example is:

when $f(0)=g(0) \wedge f(1) \neq g(1)$ ie. $f R g$

$g(0) \neq h(0) \wedge g(1)=h(1)$ ie. $g R h$.

then. we have $f(0) \neq h(0) \wedge f(1) \neq h(1)$

then. $f \not R h$.

$\therefore$ it is not transitive.

→ Since R does not have all three properties.

It is not an equivalence relation.

c) $\{(f,g) \mid f(x)-g(x)=1 \text{ for all } x\in\mathbb{Z}\}$

solution: → It is not reflexive.

$\because$ if we have $f R f$. then. we should have

$f(x)-f(x)=1$ which is impossible.

so. R is not reflexive. which ~~means~~ means $f \not R f$.

→ It is not symmetric.

$\because$ If we have $f R g$, i.e. $f(x)-g(x)=1$

then, $g(x)-f(x)=-1$, which means $g \not R f$

$\therefore$ it is not symmetric.

→ It is not transitive.

$\because$ If we have $f R g$, i.e. $f(x)-g(x)=1$

and we have $g R h$, i.e. $g(x)-h(x)=1$

$\therefore\ f(x)-g(x)+g(x)-h(x)=1+1$

$\Rightarrow\ f(x)-h(x)=2$

$\Rightarrow\ f \not R h$

$\therefore$ it is not transitive.

→ Since R does not have all three properties. then R is not an equivalence relations.

P528. 33. (8.1)

* For the following relations on the set of real numbers:

$R_1 = \{(a,b) \in \mathbb{R}^2 \mid a > b\}$, the "greater than" relation,

$R_2 = \{(a,b) \in \mathbb{R}^2 \mid a \geq b\}$, the "greater than or equal to" relation,

$R_3 = \{(a,b) \in \mathbb{R}^2 \mid a < b\}$, the "less than" relation,

$R_4 = \{(a,b) \in \mathbb{R}^2 \mid a \leq b\}$, the "less than or equal to" relation,

$R_5 = \{(a,b) \in \mathbb{R}^2 \mid a = b\}$, the "equal to" relation,

$R_6 = \{(a,b) \in \mathbb{R}^2 \mid a \neq b\}$, the "unequal to" relation.

Find:

a) $R_2 \cup R_4$

$= \mathbb{R}^2.$

b) $R_3 \cup R_6$

$= R_6$ ($\because$ R_6 includes all relations in R_3)

c) $R_3 \cap R_6$

$= R_3.$

d) $R_4 \cap R_6$

$= \{(a,b) \in \mathbb{R}^2 \mid a \leq b\} \cap \{(a,b) \in \mathbb{R}^2 \mid a \neq b\}$

$= \{(a,b) \in \mathbb{R}^2 \mid a < b\}$

$= R_3$

e) $R_3 - R_6$

$= \{(a,b) \in \mathbb{R}^2 \mid a < b\} - \{(a,b) \in \mathbb{R}^2 \mid a \neq b\}.$

$= \emptyset$

(f) $R_6 - R_3$

$= \{(a,b) \in \mathbb{R}^2 \mid a \neq b\} - \{(a,b) \in \mathbb{R}^2 \mid a < b\}$

$= \{(a,b) \in \mathbb{R}^2 \mid a > b\}$

$= R_1$

(g) $R_2 \oplus R_6$

$R_2 \oplus R_6$
$= R_2 \cup R_6 - R_2 \cap R_6$

$= \{(a,b) \in \mathbb{R}^2 \mid a \geq b\} \oplus \{(a,b) \in \mathbb{R}^2 \mid a \neq b\}$

$= \{(a,b) \in \mathbb{R}^2 \mid a \leq b\}$

$= R_4$

(h) $R_3 \oplus R_5$

$R_3 \oplus R_5$
$= R_3 \cup R_5 - R_3 \cap R_6$

$= \{(a,b) \in \mathbb{R}^2 \mid a < b\} \oplus \{(a,b) \in \mathbb{R}^2 \mid a = b\}.$

$= \{(a,b) \in \mathbb{R}^2 \mid a \leq b\}$

$= R_4$