Recitation. 10.

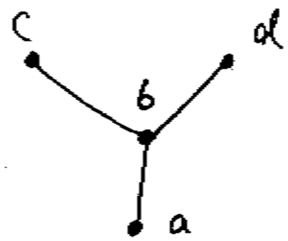
— Jie

P579. 12. Let (S,R) be a poset. show that (S,R^{-1}) is a poset, where R^{-1} is the inverse of R. The poset (S,R^{-1}) is called the dual of (S,R).

proof: This follows from the definition.

- R^{-1} is reflexive if R is.
- For antisymmetry, suppose that $(a,b) \in R^{-1}$ and $a \neq b$, Then $(b,a) \in R$, so $(a,b) \notin R$, whence $(b,a) \notin R^{-1}$.
- Finally, if $(a,b) \in R^{-1}$ and $(b,c) \in R^{-1}$, then $(b,a) \in R$ and $(c,b) \in R$, so $(c,a) \in R$ ($\because R$ is transitive). and therefore $(a,c) \in R^{-1}$; thus R^{-1} is transitive.

P579. 25. List all ordered pairs in the partial ordering with the accompanying Hasse diagram.



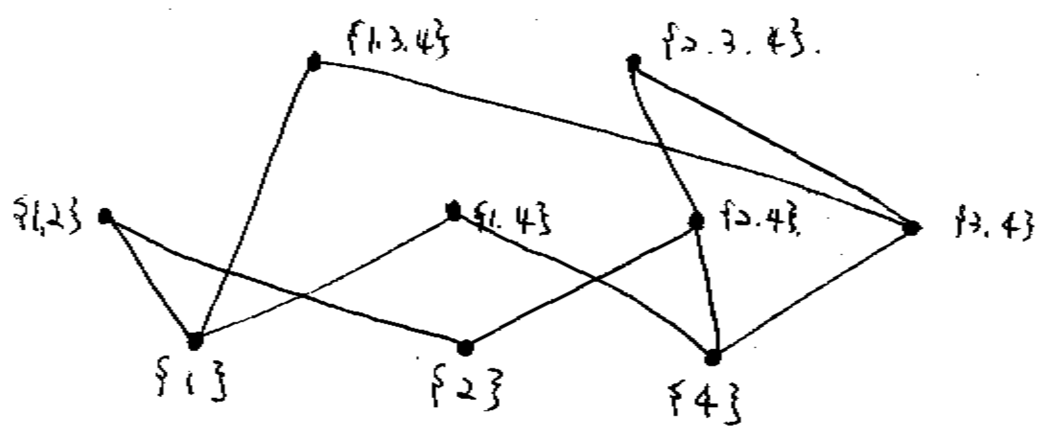
Solution:

(a, a) (b, b) (c, c) (d, d)

(a, b) (a, c) (a, d) (b, c) (b, d)

P580 35. Answer these questions from the poset $(\{\{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\}\}, \subseteq)$

① Draw the Hasse Diagram for the poset



a) Find the Maximal elements.

$\{1,2\}$, $\{1,3,4\}$, $\{2,3,4\}$

b) Find the minimal elements:

$\{1\}$, $\{2\}$, $\{4\}$

c) Is there a greatest element.

No

d) Is there a least elements.

No

e) Find all upper bounds of $\{\{2\},\{4\}\}$.

$\{2,4\}$, $\{2,3,4\}$

f) Find the least upper bound of $\{\{2\},\{4\}\}$, if it exists.

$\{2,4\}$.

g) Find all lower bounds of $\{\{1,3,4\},\{2,3,4\}\}$.

$\{3,4\}$, $\{4\}$.

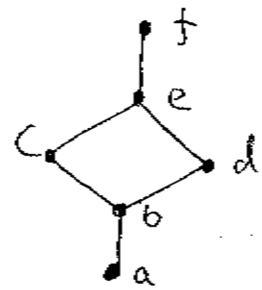
h) Find the greatest lower bound of $\{\{1,3,4\},\{2,3,4\}\}$, if it exists.

$\{3,4\}$.

P575. Example 21.

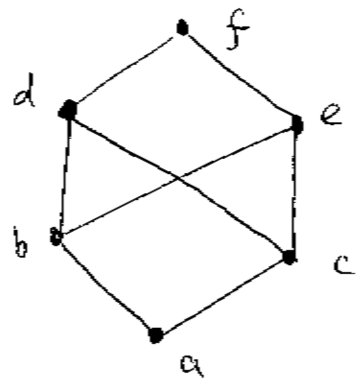
Determine whether the posets represented by each of the Hasse diagrams in Figure 8 are Lattices.

(a)



Yes. because every pair of elements has both a least upper bound and a greatest lower bound.

(b)



No, because the elements b and c have no least upper bound. ~~a. Not~~ Note that, for d. e. f, ~~e~~ each of them is an upper bound, ~~b~~ but none of these three elements precedes the other two with respect to the ordering of this poset.

P330.

3. Suppose that $(S, \preceq_1)$ and $(T, \preceq_2)$ are posets, If $\preceq$ is defined by $(s,t) \preceq (u,v)$ iff $s \preceq_1 u$, and $t \preceq_2 v$ for $s, u \in S$, $t, v \in T$, show that $(S \times T, \preceq)$ is a poset.

proof: we must show that $\preceq$ is a partial order.

① Reflexive:

$\because$ $\preceq_1$ and $\preceq_2$ are partial orders, so

$$s \preceq_1 s, \quad t \preceq_2 t,$$

Thus, $(s,t) \preceq (s,t)$.

$\therefore$ $\preceq$ is reflexive.

② Antisymmetric:

Suppose $(s,t) \preceq (u,v)$ and $(u,v) \preceq (s,t)$.

Then, $s \preceq_1 u \preceq_1 s$ and $t \preceq_2 v \preceq_2 t$

Hence, $s = u$, $t = v$

so. $(s,t) = (u,v)$

$\therefore$ $\preceq$ is antisymmetric.

③ Transitive,

Suppose $(s,t) \preceq (u,v) \preceq (w,x)$. Then

$s \preceq_1 u$, $u \preceq_1 w$, $t \preceq_2 v$ and $v \preceq_2 x$.

By transitivity of $\preceq_1$, $\preceq_2$, $s \preceq_1 w$ and $t \preceq_2 x$.

Hence, $(s,t) \preceq (w,x)$.

Thus. $\preceq$ is transitive.

$\therefore$ $\preceq$ is a partial order, $\Rightarrow$ $(S \times T, \preceq)$ is a poset.