Recti. 4:

* show $(p \vee r) \wedge (q \to r) \equiv (p \to q) \to r$.

solution: $(p \vee r) \wedge (q \to r)$ (GIVEN)

$= (p \vee r) \wedge (\neg q \vee r)$ (Implication)

$= (r \vee p) \wedge (r \vee \neg q)$ (Comutative)

$= r \vee (p \wedge \neg q)$ (Distribution)

$= (p \wedge \neg q) \vee r$ (Commutative)

$= (\neg\neg p \wedge \neg q) \vee r$ (double. Neg.)

$= \neg(\neg p \vee q) \vee r$ (De Mogen)

$= \neg(p \to q) \vee r$ (Implication)

$= (p \to q) \to r$.

* Pg. 1.2.23.

show $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are equivalence;

solution $p \to (q \wedge r)$ (GIVEN)

$= \neg p \vee (q \wedge r)$ (Implication)

$= (\neg p \vee q) \wedge (\neg p \vee r)$ (Distribution)

$= (p \to q) \wedge (p \to r)$ (Implication)

#.

* show that $((p \wedge q) \wedge r) \wedge s \equiv ((s \wedge r) \wedge q) \wedge p$

solution: $((p \wedge q) \wedge r) \wedge s$ (Given)

$\equiv s \wedge ((p \wedge q) \wedge r)$ (Commutive)

$\equiv s \wedge (r \wedge (p \wedge q))$ (Com)

$\equiv s \wedge (r \wedge (q \wedge p))$ (Com)

$= (s \wedge r) \wedge (q \wedge p)$ (Associative)

$= ((s \wedge r) \wedge q) \wedge p$ (Associative)

* show that $p \vee q \equiv \neg(\neg p \wedge \neg q)$

solution: $p \vee q \equiv \neg\neg p \vee q$ (Double neg)

$\equiv \neg\neg p \vee \neg\neg q$ (Double Neg)

$\equiv \neg(\neg p \vee \neg q)$ (De Morgen)

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1.3.33 : Express each of these statemet using quantifiers. Then form the negation of the statement. So that no negation is to the left of a gantifier. Next, express the negation in English.

a) Some old dogs can learn new tricks.

Solution: Let T(x): ~~be the~~ x can learn tricks,

~~and~~ domain: old dogs.

$\therefore$ a) $\rightarrow$ $\exists x\, T(x)$

Negation: $\neg(\exists x\, T(x))$

$\Rightarrow \forall x \neg T(x)$

no old dogs can learn new tricks.

b). No rabbit knows calculus.

Solution: C(x): x knows calculs

domain: rabbit

b) $\rightarrow$ $\neg\exists x\, C(x)$

Negation: $\neg(\neg\exists x\, C(x))$

$\Rightarrow \exists x\, C(x)$

there is a rabbit that knows calculus

e) There is no one in this class who knows French and Russian.

Solution; Let $F(x)$: x knows French

$R(x)$: x knows Russian.

domain: x: people in this class.

e) $\rightarrow \neg\exists x\ (F(x) \wedge R(x))$

Negation: $\neg(\neg\exists x\ (F(x) \wedge R(x)))$

$= \exists x\ (F(x) \wedge R(x))$

" there is someone in the class who knows French and Russian.

P58 * Mixing Quantifiers:

1.4.3 Let $Q(x,y)$ be the statement "x has sent an e-mail message to y". where the domain for both x and y consists of all student in our class.

- every body ~~send~~ has sent an email to Jie.

  $\forall x \; Q(x, \text{Jie})$

  $\forall x \; Q(\text{Jie}, x)$ --- what this mean?

- There is some student in our class who has sent a message to some student in our class.

  $\exists x \; \exists y \; Q(x, y)$

- There is some body whom Jie does not send an email to.

  $\exists x \; \neg Q(\text{Jie}, x)$

1.4:3

- There is some student in our class who has sent a message to every student in our class.

b): $\exists x \forall y \; Q(x, y)$

- what's the meaning of the following expressions?

c) $\forall x \exists y \; Q(x, y)$ SentMess(x, y)

same? d) $\exists y \forall x \; Q(x, y)$.

No ! very different!

(c): Every student in our class has sent a message to at least one student in our class

(d) There is (exist) a student in our class who has been sent a message by every student in our class

— binding variables.

— negation: $\neg \forall x \; P(x) \equiv \exists x \; \neg P(x)$

$\neg \exists x \; P(x) \equiv \forall x \; \neg P(x)$

1.4.9. Let : $L(x,y)$: be the statement "x loves y"

Universe of discourse x ~~x~~ y consists of all people in the world.

Use quantifiers to express each of these statements.

a) Everybody loves Jerry

$$\forall x \ L(x, \text{Jerry})$$

b) Everybody loves somebody.

$$\forall x \exists y \ L(x,y)$$

c) There is somebody whom everybody loves.

$$\exists y \forall x \ L(x,y)$$

d) Nobody loves everybody.

$$\forall x \exists y \ \neg L(x,y)$$

e) There is somebody whom Lydia does not love

$$\exists x \left(\neg L(\text{Lydia}, x) \right)$$

f) There is somebody whom no one loves.

$$\exists x . \forall y \ \neg L(y,x)$$

g) There is exactly one person whom ~~ev~~ everybody loves.

$$\exists x \left(\forall y \ L(y,x) \land \forall z \left((\forall w \ L(w,z)) \to z = x \right) \right)$$

h) There are exactly two people whom Lynn loves.

$$\exists x \exists y \left(x \neq y \land L(\text{Lynn}, x) \land L(\text{Lynn}, y) \land \forall z \left(L(\text{Lynn}, z) \to (z = x \lor z = y) \right) \right).$$