Title: Informed Search Methods

Required reading: AIMA, Chapter 4. LWH: Chapter 13 and 14.

Introduction to Artificial Intelligence
CSCE 476-876, Spring 2002

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Glossary: Given: Monday, Feb 18, 2002. Due: Mon, Feb 25, 2002

$g(n), h(n), f(n), f^*$

Admissible heuristic

Constructive search

Effective branching factor

Evaluation function

Evaluation function: (1) Monotonic, (2) Myopic

Gradient ascent

Gradient descent

Heuristic

Hill climbing

Iterative improvement

Iterative repair

Manhattan distance

Locally finite graphs

Optimally efficient (for a given heuristic function)

Pathmax equation

Random restart

Relaxed problem

Search: (1) Ordered search, (2) Best-first search

Simulated annealing

Outline

- Categorization of search techniques
- Ordered search (search with an evaluation function)
- Best-first search:
  (1) Greedy search (2) A*
- Admissible heuristic functions:
  how to compare them?
  how to generate them?
  how to combine them?
- Iterative improvement search:
  (1) Hill-climbing (2) Simulated annealing

Types of Search (I)

1- Uniformed vs. informed
2- Systematic/constructive vs. iterative improvement

Uniformed :

use only information available in problem definition,
no idea about distance to goal
→ can be incredibly ineffective in practice

Heuristic :

exploits some knowledge of the domain
also useful for solving optimization problems

Types of Search (II)

Systematic, exhaustive, constructive search:
a partial solution is incrementally extended into global solution

Partial solution =
sequence of transitions between states

Global solution =
Solution from the initial state to the goal state

Examples: { Uninformed
Informed (heuristic): Greedy search, A*

→ Returns the path; solution = path

Types of Search (III)

Iterative improvement:
A state is gradually modified and evaluated until
reaching an (acceptable) optimum

→ We don't care about the path, we care about 'quality' of state
→ Returns a state; a solution = good quality state
→ Necessarily an informed search

Examples (informed): { Hill climbing
Simulated Annealing (physics), Taboo search
Genetic algorithms (biology)

Ordered search

• Strategies for systematic search are generated by choosing which node from the fringe to expand first

• The node to expand is chosen by an **evaluation function**, expressing 'desirability' $\longrightarrow$ **ordered search**

• When nodes in queue are sorted according to their decreasing values by the evaluation function $\longrightarrow$ **best-first search**

• Warning: 'best' is actually 'seemingly-best' given the evaluation function. Not always best (otherwise, we could march directly to the goal!)

Search using an evaluation function

• Example: uniform-cost search!

What is the evaluation function?

Evaluates cost from ..... to .....?

• How about the cost **to** the goal?

$h(n)$ = estimated cost of the cheapest path from the state at node n to a goal state

$h(n)$ would help focusing search

Best-first search

1. Greedy search chooses the node n closest to the goal such as $h(n)$ is minimal

2. A* search chooses the least-cost solution

   solution cost $f(n)$ $\begin{cases} g(n)\text{: cost from root to a given node } n \\ + \\ h(n)\text{: cost from the node } n \text{ to the goal node} \end{cases}$

   such as $f(n) = g(n) + h(n)$ is minimal

Greedy search (I)

→ First expand the node whose state 'closest' to the goal!

→ Minimize $h(n)$

function BEST-FIRST-SEARCH(*problem,* EVAL-FN) **returns** a solution sequence
inputs: *problem*, a problem
Eval-Fn, an evaluation function

Queueing-Fn ← a function that orders nodes by EVAL-FN
return GENERAL-SEARCH(*problem, Queueing-Fn*)

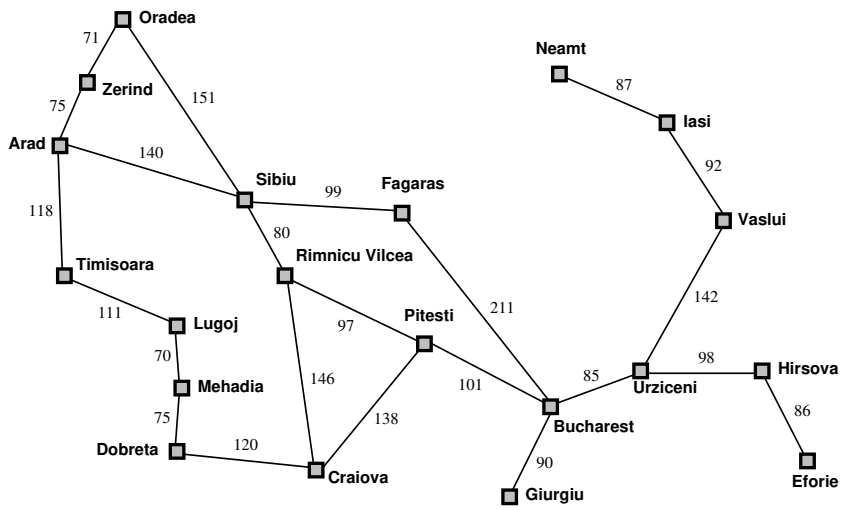
→ Usually, cost of reaching a goal may be estimated, not determined exactly

→ If state at n is goal, $h(n)$= ?

→ How to choose $h(n)$? Problem specific! Heuristic!

Greedy search (II) Romania

$h_{\text{SLD}}(n)$ = straight-line distance between n and goal location



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
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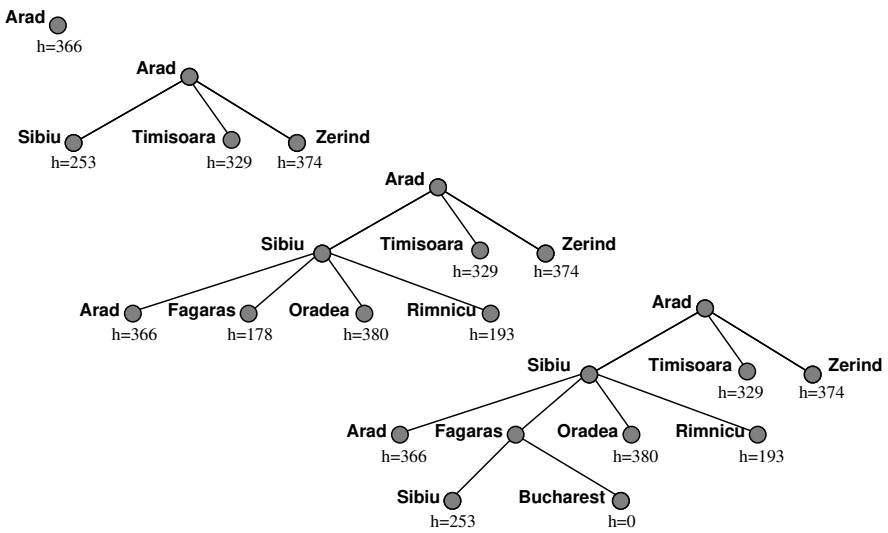
Urziceni	80
Vaslui	199
Zerind	374

Greedy search (III) Romania

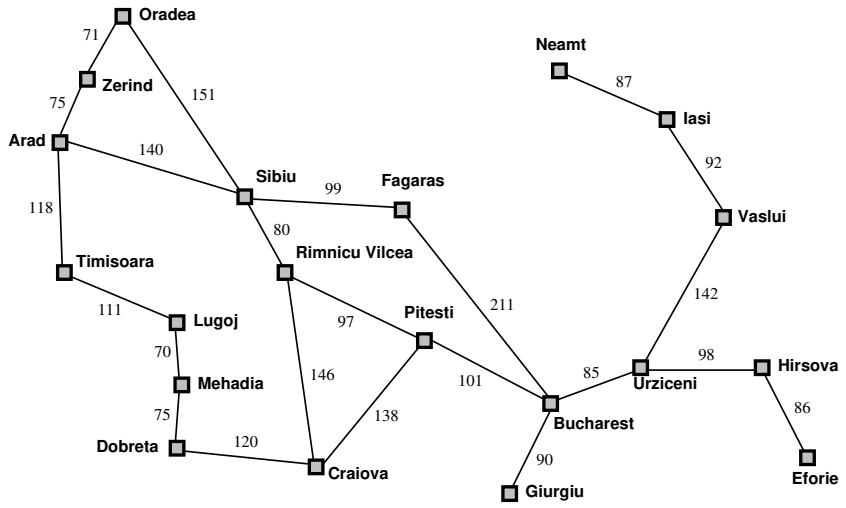
Trip from Arad to Bucharest



... Greedy search! quick, but not optimal!

Greedy search (IV) Problems

From Iasi to Fagaras?
- False starts: Neamt is a dead-end
- Looping



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Greedy search (V) Properties

→ Like depth-first, tends to follow a single path to the goal

→ Like depth-first
- Not complete
- Not optimal

→ Time complexity: $O(b^m)$, m maximum depth

→ Space complexity: $O(b^m)$ retains all nodes in memory

→ Good h function (considerably) reduces space and time
but h functions are problem dependent :—(

Hmm...

Greedy search minimizes estimated cost to goal $h(n)$

→ cuts search cost considerably

→ but not optimal, not complete

Uniform-cost search minimizes cost of the path so far $g(n)$

→ is optimal and complete

→ but can be wasteful of resources

New-Best-First search minimizes $f(n) = g(n) + h(n)$

→ combines greedy and uniform-cost searches

$f(n)$ = estimated cost of cheapest solution via n

→ Provably: complete and optimal, if $h(n)$ is admissible

A* Search (I)

$$\left.\begin{array}{l}\text{Best-first search}\\ h(n)\text{ admissible}\end{array}\right\} \longrightarrow \text{A}^* \text{ search, provably } \left\{\begin{array}{l}\text{optimal}\\ \text{complete}\end{array}\right.$$

Admissible heuristic

An admissible heuristic is a heuristic that never overestimates the cost to reach the goal

→ is optimistic

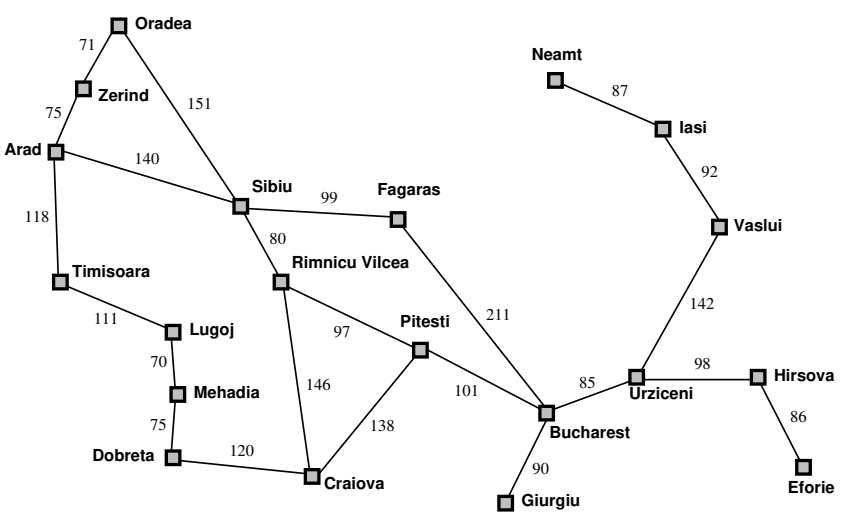
→ thinks the cost of solving is less than it actually is

Example:
- travel: straight line distance
- I can finish college in 3 years
- We can fly to Mars by 2003

If h is admissible,

$f(n)$ never overestimates the actual cost of the best solution through n.

A* Search (II) Romania: from Sibiu to Bucharest



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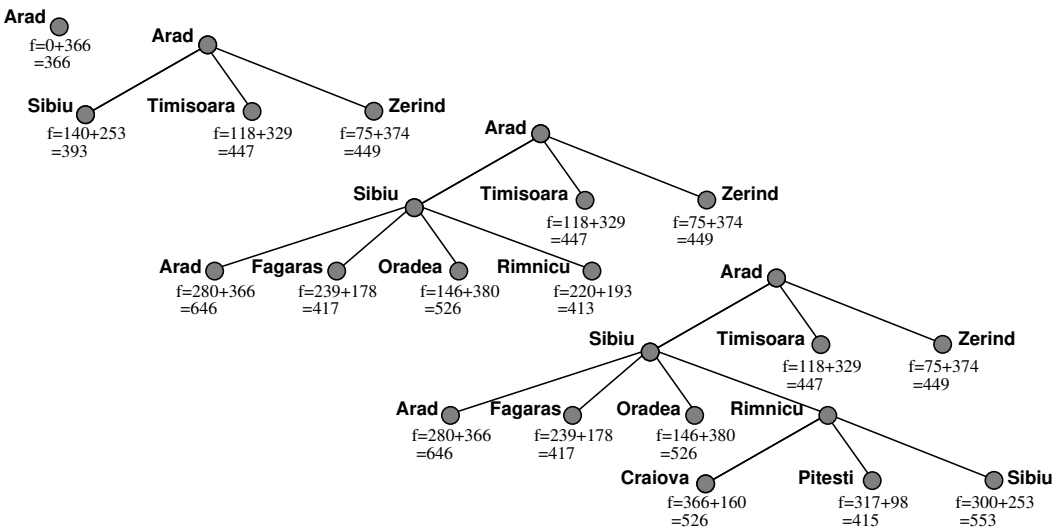
A* Search (III) Romania: from Sibiu to Bucharest



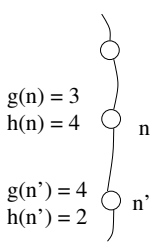
A* Search (IV) Observation: pathmax

Monotonicity: f-cost never decreases

This holds for almost all admissible heuristics

If f is not monotonic, use **pathmax** equation

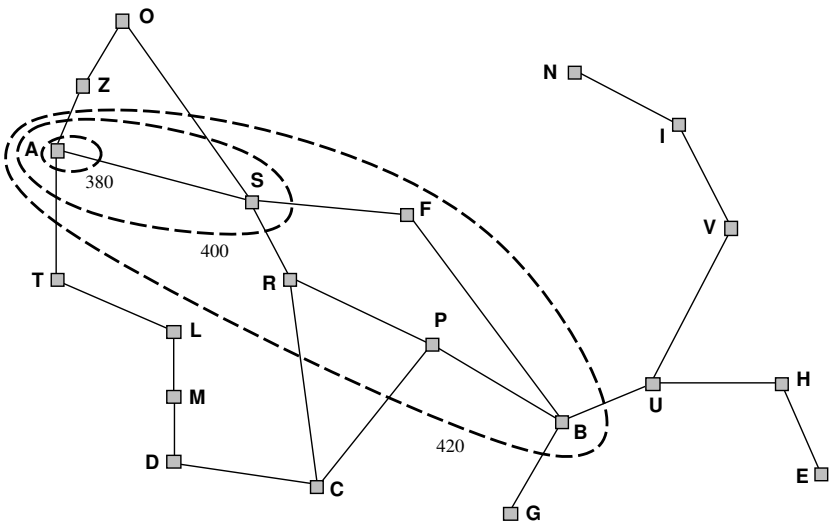
$$f(n') = max(f(n), g(n') + h(n'))$$



A* never decreasesalong any path out from root

$\longrightarrow$ we can conceptually draw contours in the search space

A* Search (V) Romania: from Sibiu to Bucharest



A* Search (VI) Observations

We define f^*: cost of optimal solution

A* search
- expands all nodes with $f(n) < f^*$
- h admissible, thinks the cost of solving $<$ it actually is
- may expand nodes on the 'goal-contour' $f(n) = f^*$
- before selecting goal

A* is optimally efficient, optimal, complete

A* **Search** (VII) is optimally efficient

.. for any given evaluation function: no other algorithms that finds the optimal solution is guaranteed to expend fewer nodes than A*

Interpretation (proof not presented): Any algorithm that does not expand all nodes between root and the goal contour risks missing the optimal solution

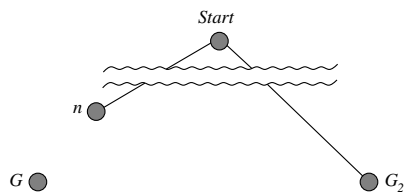
A* **Search** (VIII) is optimal

G, G_2 goal states $\Rightarrow g(G) = f(G)$, $f(G_2) = g(G_2)$ $\quad h(G) = h(G_2) = 0$

G optimal goal state $\Rightarrow f^* = f(G)$

G_2 suboptimal $\Rightarrow f(G_2) > f^* = f(G)$ (1)

Suppose n is not chosen for expansion



h admissible $\Rightarrow f^* \geq f(n)$ (2)

Since n was not for expansion $\Rightarrow f(n) \geq f(G_2)$ (3)

(2) + (3) $\Rightarrow f^* \geq f(G_2)$ (4)

(1) and (4) are contradictory $\Rightarrow n$ should be chosen for expansion

A* **Search** (IX) is complete

Since A* search expands all nodes with $f(n) < f^*$, it must eventually reach the goal state unless there are infinitely many

nodes $f(n) < f^*$ {
1. $\exists$ a node with infinite branching factor
or
2. $\exists$ a path with infinite number of nodes along it

A* is complete if {
on locally finite graphs
and
$\exists \delta > 0$ constant, the cost of each operator $> \delta$

A* **Search** (X) Complexity

Time :

Exponential in (relative error in h $\times$ length of solution path)

... quite bad

Space : must keep all nodes in memory

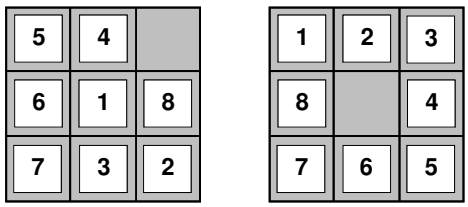
major drawback of A*

$\rightarrow$ Memory Bounded Search in Section 4.3 (not addressed here)

Admissible Heuristic functions (I)

Examples

- Route-finding problems: straight-line distance
- 8-puzzle: $\begin{cases} h_1(n) = \text{number of misplaced tiles} \\ h_2(n) = \text{total Manhattan distance} \end{cases}$



$h_1(S) = ?$
$h_2(S) = ?$

Admissible heuristic functions (II)

Performance

Two criteria to compare admissible heuristic functions:

1. Effective branching factor: b^*
2. Dominance: number of nodes expanded

Effective branching factor b^*

– The heuristic expands N nodes in total

– The solution depth is d

$\longrightarrow$ b^* is the branching factor had the tree been uniform

$$N = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d = \frac{(b^*)^{d+1} - 1}{b^* - 1}$$

– Example: N=52, d=5 $\rightarrow$ $b^* = 1.91$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs: nodes expanded

Sol. depth	**IDS**	**A**$^*(h_1)$	**A**$^*(h_2)$
$d = 14$	3,473,941	539	113
$d = 24$	too many	39,135	1,641

A* expands all nodes $f(n) < f^* \Rightarrow g(n) + h(n) < f^*$
$\Rightarrow h(n) < f^* - g(n)$

If $h_1 \leq h_2$, A* with h_1 will always expand at least as many (if not more) nodes than A* with h_2

$\longrightarrow$ It is always better to use a heuristic function with higher values, as long as it does not overestimate (remains admissible)

Admissible heuristic functions (III)

How to generate admissible heuristics?

→ Use *exact* solution cost of a relaxed (easier) problem

Steps:
– Consider problem P
– Take a problem P' easier than P
– Find solution to P'
– Use solution of P' as a heuristic for P

- If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*: the shortest solution can be used as a heuristic ($\equiv h_1(n)$)
- If the rules are relaxed so that a tile can move to *any adjacent square*: the shortest solution can be used as a heuristic ($\equiv h_2(n)$)
- For TSP: let path be *any* structure that connects all cities
  $\Longrightarrow$ minimum spanning tree heuristic (polynomial)

Admissible heuristic functions (IV)

We have a set of admissible heuristics $h_1, h_2, h_3, \ldots, h_m$ but no heuristic that dominates all others, what to do?

$$\longrightarrow h(n) = \texttt{max}(h_1(n), h_2(n), \ldots, h_m(n))$$

→ Problem:
Cost of computing the heuristic (vs. cost of expanding nodes)

Iterative improvement (a.k.a. local search)

⟶ Sometimes, the 'path' to the goal is irrelevant
only the state description (or its quality) is needed

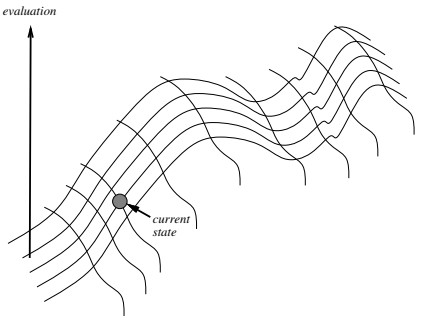
e.g., a CSP:
- Are all constraints satisfied?
- Is a maximum # of constraints satisfied?
- Is a maximum # of important constraints satisfied?

Iterative improvement search:
gradually modify a sub-optimal state
until reaching a near-optimal state

Example: complete-state formulation of N-queens

Intuition

All states are layed up on the surface of a landscape
A state quality/value is its height
Move around and try to reach the highest peak



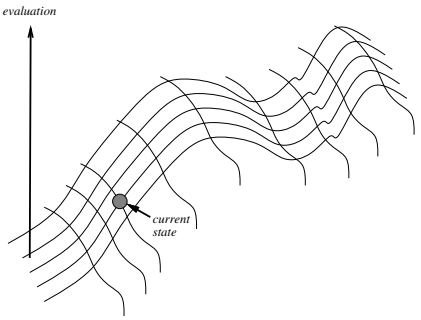
Two major classes:
- Hill climbing (a.k.a. gradient ascent/descent)
  → try to make changes to improve current state
- Simulated Annealing (physics)
  → things can temporarily get worse

Hill climbing

Start from any state at random and loop:

Examine all direct neighbors

If a neighbor has higher value then move to it else exit



Problems:
- Local maxima: search halts
- Plateau (flat): random walk
- Ridge: may oscillate from side to side, slow progress

Random-restart hill-climbing

→ When HC halts or no progress is made
re-start from a different (randomly chosen) starting
save best results found so far

→ Repeat random restart
- for a fixed number of iterations, or
- until best results have not been improved for a certain number of iterations

⟶ Optimality (soundness)? Completeness?
⟶ Complexity: space? time?

⟶ In practice, surprisingly good.. (eroding myth)

Simulated annealing (I)

Basic idea: When stuck in a local maximum allow few steps towards less good neighbors to escape the local maximum

Start from any state at random, start count down and loop until time is over:

Pick up a neighbor at random

Set ΔE = value(neighbor) - value(current state)

If ΔE>0 (neighbor is better)

then move to neighbor & restart count down,

else ΔE<0 move to it with probability < 1

Transition probability $\simeq e^{\Delta E/T}$ $\left\{ \begin{array}{l} \Delta\text{E is negative} \\ \text{T: count-down time} \end{array} \right.$

as time passes, less and less likely to make the move towards 'unattractive' neighbors

Simulated annealing (II)

Analogy to physics:

Gradually cooling a liquid until it freezes

If temperature is lowered sufficiently slowly, material will attain lowest-energy configuration (perfect order)

Count down	$\longleftrightarrow$	Temperature
Moves between states	$\longleftrightarrow$	Thermal noise
Global optimum	$\longleftrightarrow$	Lowest-energy configuration

How about decision problems?

Optimization problems		**Decision problems**
Iterative improvement	$\longleftrightarrow$	Iterative repair
State value	$\longleftrightarrow$	Number of constraints violated
Sub-optimal state	$\longleftrightarrow$	Inconsistent state
Optimal state	$\longleftrightarrow$	Consistent state