Title: Solving Problems by Searching
AIMA: Chapter 3 (Sections 3.4, 3.5, and 3.6)

Introduction to Artificial Intelligence
CSCE 476-876, Spring 2002
URL: `www.cse.unl.edu/~choueiry/S02-476-876`

Berthe Y. Choueiry (Shu-we-ri)
`choueiry@cse.unl.edu`, `(402)472-5444`

List of terms for Glossary 3:

Given: Mon, Feb 11, 2001

Due date: Mon, Feb 18, 2001

- Completeness
- Complexity (time/space)
- Diameter (of a graph). *Look in graph theory.*
- Expand (a search node)
- Fringe
- Optimality
- Search:
  - Breadth-first
  - Heuristic
  - Node
  - Space
  - Strategy
  - Tree
  - Uniformed (blind)

Summary

- Problem-solving agents

  Formulate, Search, Execute

- Formulating problems
  - Problem types:
    Simple-state, Multiple-State, Contingency, Exploration
  - Problem components: State, Operators, Goal test, Path cost
    Solution: sequence of actions from initial state to goal state
  - Importance of modeling

Today:

$\longrightarrow$ Search

Outline

- Generate action sequences
  - tree, node, fringe, queue.
  - Data structure
- Search strategies:
  - Four evaluation criteria.. ?
  - Six strategies for uniformed search

Search: generate action sequences

partial solution: sequence yielding a (non goal) intermediate state

$$\text{Search} \left\{ \begin{array}{c} \text{generate} \\ \text{maintain} \end{array} \right\} \text{a set of sequences of partial solutions}$$

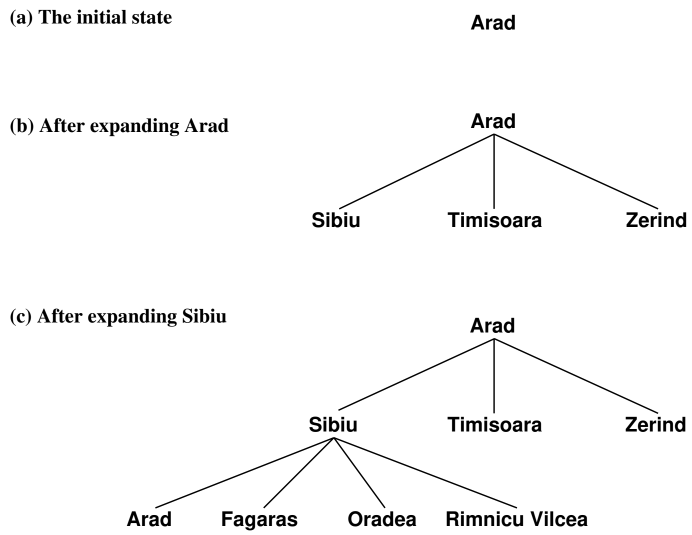
Two aspects:

1. how to generate sequences
2. which data structures to keep track of them

Search generate action sequences

Basic idea:
offline, simulated exploration of state space
by generating successors of already-explored states
$\rightarrow$ *expanding* states

Start from a state, test if it is a goal state
If it is, we are done
If it is not: *expand state*
Apply all operators applicable to current state to
generate all possible sequences of future states
now we have set of partial solutions
...



Search tree, nodes
- root: initial state
- leaves: states that can/should not be expanded

Data structure

LHW Chapter 13

for a search node
- state in space state
- parent node
- operator used to generate node
- **depth**: # nodes from root (path length)
- path cost $g(x)$

A node x has a parent, children, depth, path cost $g(x)$

Nodes to be expanded
- constitute a fringe (frontier)
- managed in a queue

Warning:

Do not confuse: State space and Search (tree) space

Holiday in Romania: What is a state?
What is the state space?
What is the size of state space?

What is the size of search tree ?

A node has a parent, children, depth, path cost $g(x)$

A state has no parent, children, depth, etc..


```
function GENERAL-SEARCH(problem, strategy) returns a solution, or failure
   initialize the search tree using the initial state of problem
   loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
   end
```

Essence of search: which node to expand first?
$\longrightarrow$ search strategy

A strategy is defined by picking the *order of node expansion*

Types of Search

Uniformed : use only information available in problem definition

Heuristic : exploits some knowledge of the domain

Uninformed search strategies

1. Breadth-first search
2. Uniform-cost search
3. Depth-first search
4. Depth-limited search
5. Iterative deepening search
6. Bidirectional search

Search strategies

Criteria for evaluating search:

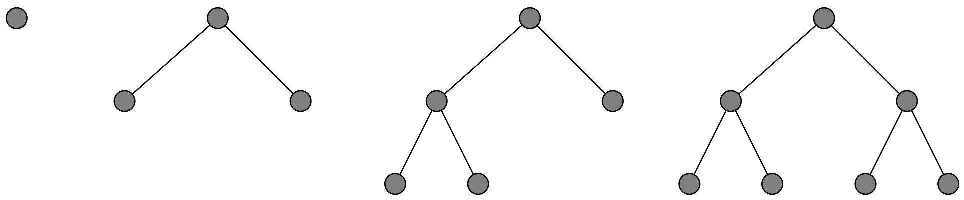
1. Completeness: does it always find a solution if one exists?
2. Time complexity: number of nodes generated/expanded
3. Space complexity: maximum number of nodes in memory
4. Optimality: does it always find a least-cost solution?

Time/space complexity measured in terms of:

- b: maximum branching factor of the search tree
- d: depth of the least-cost solution
- m: maximum depth of the state space (may be ∞)

Breadth-first search (I)

→ Expand root node
→ Expand *all* children of root
→ Expand *each* child of root
→ Expand successors of each child of root, etc.



⟶ Expands nodes at depth d before nodes at depth $d+1$
⟶ Systematically considers all paths length 1, then length 2, etc.
⟶ Implement: put successors at end of queue

Breadth-first search (II)

⟶ One solution?
⟶ Many solutions? Finds shallowest goal first

1. Complete?
2. Optimal? (provided cost increases monotonically with depth)
3. Time? $O(b^d)$ $\begin{cases} \text{branching factor } b \\ \text{depth } d \end{cases}$
4. Space? same, $O(b^d)$, keeps every node in memory, big problem

Depth	Nodes	Time	Memory
0	1	1 millisecond	100 bytes
2	111	.1 seconds	11 kilobytes
4	11,111	11 seconds	1 megabyte
6	10^6	18 minutes	111 megabytes
8	10^8	31 hours	11 gigabytes
10	10^{10}	128 days	1 terabyte
12	10^{12}	35 years	111 terabytes
14	10^{14}	3500 years	11,111 terabytes

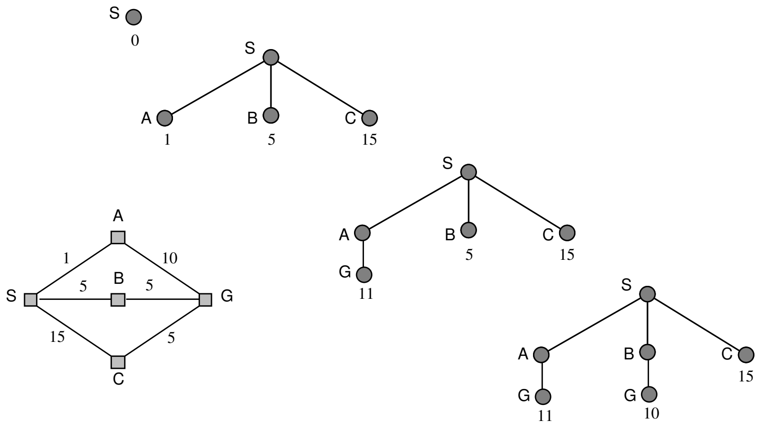
Uniform-cost search (I)

$\longrightarrow$ Breadth-first does not consider path cost $g(x)$

$\longrightarrow$ Uniform-cost expands first lowest-cost node on the fringe

$\longrightarrow$ Implement: sort queue in decreasing cost order

When $g(x) = \text{Depth}(x) \longrightarrow$ Breadth-first $\equiv$ Uniform-cost



(a) (b)

Uniform-cost search (II)

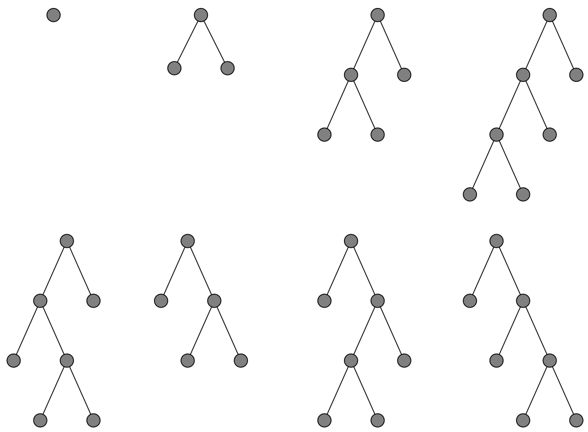
1. Complete?
   Yes, if cost $\geq \epsilon$
2. Optimal?
   If the cost is a monotonically increasing function
   When cost is added up along path, an operator's cost ....?
3. Time?
   \# of nodes with $g \leq$ cost of optimal solution
   worst-case $O(b^d)$
4. Space?
   \# of nodes with $g \leq$ cost of optimal solution
   worst-case $O(b^d)$

Depth-first search (I)

⟶ Expands nodes at deepest level in tree

⟶ When dead-end, goes back to shallower levels

⟶ Implement: put successors at end of queue (or, recursive calls)



⟶ Little memory: path and unexpanded nodes

For b: branching factor, m: maximum depth, space ....?

Depth-first search (II)

Time complexity:

We may need to expand all paths, $O(b^m)$

When there are many solutions, DFS may be quicker than BFS

When m is big, ∞ (deep, loops), .. troubles

⟶ Major drawback of DFS: going deep where there is no solution..

Properties:

1. Complete?
2. Optimal?
3. Time? Woow.
4. Space?

Depth-limited search (I)

⟶ DFS is going too deep, put a threshold on depth!
For instance, 20 cities on map for Romania, any node deeper than 19 is cycling. Don't expand deeper!

⟶ Implement: nodes at depth l have no successor

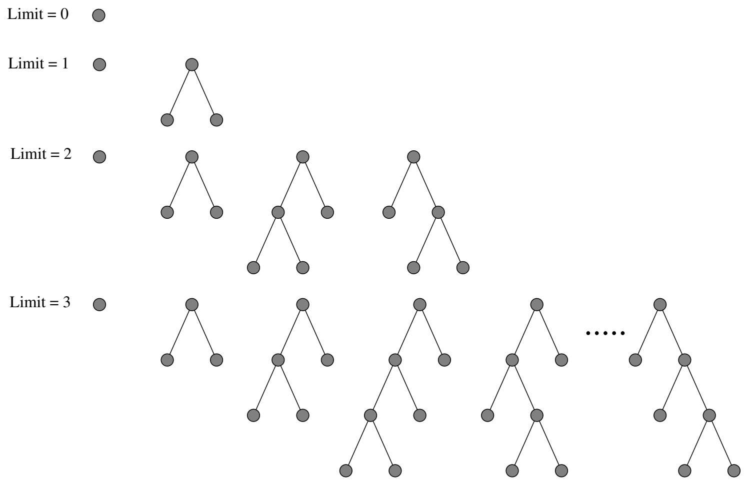
Properties:

1. Complete?
2. Optimal?
3. Time? (given l depth limit)
4. Space? (given l depth limit)

Problem: how to choose l?

Iterative-deepening search (I)

→ DLS with depth = 0
→ DLS with depth = 1
→ DLS with depth = 2
→ DLS with depth = 3...



⟶ Combines benefits of DFS and BFS

Iterative-deepening search (II)

$\longrightarrow$ combines benefits of DFS and BFS

Properties:

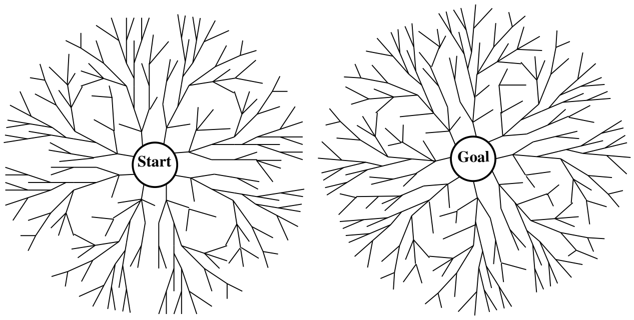
1. Complete? like BFS
2. Optimal? like BFS (if step cost = 1)
3. Space? like DFS
4. Time? same

$\longrightarrow$ Some nodes are expanded several times, wasteful?
 # expansions in BFS: $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$
 # in IDS: $(d+1).1 + (d).b + (d-1).b^2 + \ldots + 1.b^d = O(b^d)$

$\longrightarrow$ IDS is preferred when search space is large and depth unknown

Bidirectional search (I)

$\rightarrow$ Given initial state and the goal state, start search from both ends and meet in the middle



$\rightarrow$ Assume same b branching factor, $\exists$ solution at depth d, time: $O(2b^{d/2}) = O(b^{d/2})$

$b = 10, d = 6$, DFS= 1,111,111 nodes, BDS=2,222 nodes!

Bidirectional search (II)

In practice :—(

- Need to define predecessor operators to search backwards
  If operator are invertible, no problem
- What if $\exists$ many goals (set state)?
  do as for multiple-state search
- need to check the 2 fringes to see how they match
  need to check whether any node in one space appears in the other space (use hashing)
  need to keep all nodes in a half in memory $O(b^{d/2})$
- What kind of search in each half space?

Summary

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

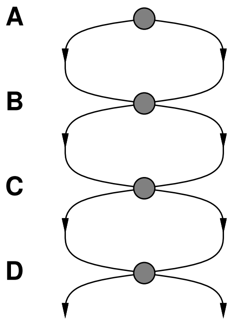
b branching factor
d solution depth
m maximum depth of tree
l depth limit

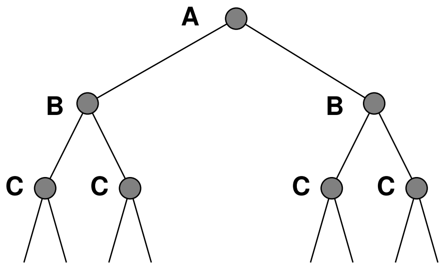
Loops: Avoid repeated states (I)

Avoid expanding states that have already been visited

Valid for both infinite and finite trees

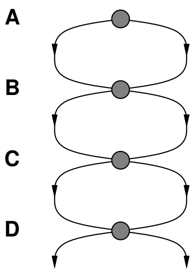
$$\text{Example: } \begin{cases} m \text{ maximum depth} \\ m+1 \text{ states} \\ 2^m \text{ possible branches (paths)} \end{cases}$$

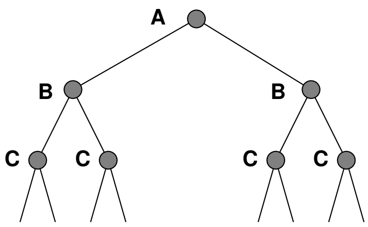




Loops: (II)

$$\text{3 Strategies:} \begin{cases} \text{1. Do not return to the state you come from} \\ \text{2. Do not return to any of ancestor state} \\ \text{3. Do not return to any previously visited state} \\ \quad \text{...... required space?} \end{cases}$$





$\longrightarrow$ Implementation: hash table.

Trade-off cost of storing+checking vs. cost of searching

Summary

Path: sequence of actions leading from one state to another

Partial solution: a path from an initial state to another state

Search: develop a sets of partial solutions

- Search tree & its components (node, root, leaves, fringe)
- Data structure for a search node
- Search space vs. state space
- Node expansion, queue order
- Search types: uniformed vs. heuristic
- 6 uniformed search strategies
- 4 criteria for evaluating & comparing search strategies