Title: First-Order Logic
AIMA: Chapter 7 (Sections 7.1 and 7.3)

Introduction to Artificial Intelligence
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Outline

- First-order logic:
  - basic elements
  - syntax
  - semantics
- Examples

Propositional Logic

- is simple
- illustrates important points:
  model, inference, validity, satisfiability, ..
- is restrictive: world is a set of facts
- lacks expressiveness

First-Order Logic

- more symbols (objects, properties, relations)
- more connectives (quantifier)

First Order Logic

→ provides more "primitives" to express knowledge:
— objects (identity & properties)
— relations among objects (including functions)

Objects: people, houses, numbers, Einstein, Huskers, event, ..
Properties: smart, nice, large, intelligent, loved, occurred, ..
Relations: brother-of, bigger-than, part-of, occurred-after, ..
functions: father-of, best-friend, double-of, ..

Examples: (objects? function? relation? property?)
— one plus two equals four [sic]
— squares neighboring the wumpus are smelly

Logic

Attracts: mathematicians, philosophers and AI people

Advantages:
— allows to represent the world and reason about it
— expresses anything that can be programmed

Non-committal to:
— symbols could be objects or relations
(*e.g.*, King(Gustave), King(Sweden, Gustave))
— classes, categories, time, events, uncertainty

.. but amenable to extensions: OO FOL, temporal logics, situation/event calculus, modal logic, etc.

$\longrightarrow$ Some people think FOL *is* the language of AI
true/false? donno :—(but it will remain around for some time..

Basic elements

In propositional logic, every expression is a sentence

In FOL,

- Terms
- Sentences:
  - atomic sentences
  - complex sentences
- Quantifiers:
  - Universal quantifier
  - Existential quantifier

Term

logical expression that refers to an object

— built with: constant symbols, variables, function symbols

$$\text{Term} = \mathit{function}(\mathit{term}_1, \ldots, \mathit{term}_n) \text{ or constant or variable}$$

— **ground term**: term with no variable

Atomic sentences

state facts

built with terms and predicate symbols

$$\text{Atomic sentence} = \mathit{predicate}(\mathit{term}_1, \ldots, \mathit{term}_n) \text{ or } \mathit{term}_1 = \mathit{term}_2$$

Examples:

Brother (Richard, John)

Married (FatherOf(Richard), MotherOf(John))

Complex Sentences

built with atomic sentences and logical connectives

$\neg S$
$S_1 \wedge S_2$
$S_1 \vee S_2$
$S_1 \Rightarrow S_2$
$S_1 \Leftrightarrow S_2$

Examples:
Sibling(KingJohn,Richard) $\Rightarrow$ Sibling(Richard,KingJohn)
$>(1,2) \vee \leq(1,2)$
$>(1,2) \wedge \neg>(1,2)$

Truth in first-order logic

Sentences are true with respect to a <u>model</u> and an <u>interpretation</u>

Model contains objects and relations among them

Interpretation specifies referents for
constant symbols $\rightarrow$ <u>objects</u>
predicate symbols $\rightarrow$ <u>relations</u>
function symbols $\rightarrow$ <u>functional relations</u>

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the <u>objects</u> referred to by $term_1, \ldots, term_n$
are in the <u>relation</u> referred to by *predicate*

Quantifiers

allow to make statements about entire collections of objects

- universal quantifier: make statements about everything
- existential quantifier: make statements about some things

Universal quantification

$\forall \langle variables \rangle \ \langle sentence \rangle$

Example: all dogs like bones $\forall\, x Dog(x) \Rightarrow LikeBones(x)$
x = Indy is a dog x = Indiana Jones is a person

$\forall\, x\, P$ is equivalent to the conjunction of instantiations of P

$$\begin{aligned} & Dog(Indy) \Rightarrow LikeBones(KingJohn) \\ \wedge\quad & Dog(Rebel) \Rightarrow LikeBones(Rebel) \\ \wedge\quad & Dog(Rover) \Rightarrow LikeBones(Rover) \\ \wedge\quad & \ldots \end{aligned}$$

Typically: $\Rightarrow$ is the main connective with $\forall$
Common mistake: using $\wedge$ as the main connective with $\forall$
Example: $\forall\, x \ \ Dog(x) \ \wedge \ LikeBones(x)$
all objects in the world are dogs, and all like bones

Existential quantification

$\exists \langle variables \rangle \ \langle sentence \rangle$

Example: some top-student will attend AAAI
$\exists\, x TopStudent(x) \land AttendsAAAI(x)$
Pat, Leslie, Chris are top-students

$\exists\, x\, P$ is equivalent to the disjunction of instantiations of P

$$
\begin{array}{ll}
 & TopStudent(Pat) \land AttendsAAAI(Pat) \\
\lor & TopStudent(Leslie) \land AttendsAAAI(Leslie) \\
\lor & TopStudent(Chris) \land AttendsAAAI(Chris) \\
\lor & \ldots
\end{array}
$$

Typically: $\land$ is the main connective with $\exists$
Common mistake: using $\Rightarrow$ as the main connective with $\exists$
$\exists\, x\, TopStudent(x) \ \Rightarrow \ AttendsAAAI(x)$
is true if there is anyone who is not TopStudent

Properties of quantifiers (I)

$\forall x\, \forall y$ is the same as $\forall y\, \forall x$

$\exists x\, \exists y$ is the same as $\exists y\, \exists x$

$\exists x\, \forall y$ is not the same as $\forall y\, \exists x$

$\exists x\, \forall y\, Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall y\, \exists x Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall x\, Likes(x, IceCream) \qquad \neg\, \exists x\, \neg Likes(x, IceCream)$

$\exists x\, Likes(x, Broccoli) \qquad \neg\, \forall x\, \neg Likes(x, Broccoli)$

Parsimony principal: $\forall$, $\neg$, and $\Rightarrow$ are sufficient

Properties of quantifiers (II)

Nested quantifier:
$\forall\, x(\exists\, y(P(x,y))$:
every object in the world has a particular property, which is the property to be related to some object by the relation P

$\exists\ x\ (\forall\, y(P(x,y))$:
there is some object in the world that has a particular property, which is the property to be related to every object by the relation P

Lexical scoping: $\forall\, x[Cat(x) \vee \exists\, xBrother(Richard,x)]$

Well-formed formulas (WFF): (kind of correct spelling)
every variable must be introduced by a quantifier
$\forall\, xP(y)$ is not a WFF

Examples

Brothers are siblings

.

"Sibling" is reflexive

.

One's mother is one's female parent

.

A first cousin is a child of a parent's sibling

Examples

.

$\forall x, y \; Brother(x,y) \;\Leftrightarrow\; Sibling(x,y)$

.

$\forall x, y \; Sibling(x,y) \;\Leftrightarrow\; Sibling(y,x)$

.

$\forall x, y \; Mother(x,y) \;\Leftrightarrow\; (Female(x) \;\wedge\; Parent(x,y))$

.

$\forall x, y \; FirstCousin(x,y) \Leftrightarrow$
$\exists a, b \; Parent(a,x) \wedge Sibling(a,b) \wedge Parent(b,y)$

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

predicate, terms, and equality to form atomic sentences

Father(John)=Henry

AIMA, Exercise 7.4 & 7.5

Useful to distinguish two objects: Spot has at least two sisters

.

Knowledge representation (KR)

Domain: a section of the world about which we wish to express some knowledge

Example: Family relations (kinship):
– Objects: people
– Properties: gender, married, divorced, single, widowed
– Relations: parenthood, brotherhood, marriage..

Unary predicates: Male, Female
Binary relations: Parent, Sibling, Brother, Child, etc.
Functions: Mother, Father

$\forall\, m, c, Mother(c) = m \;\Leftrightarrow\; Female(m) \wedge Parent(m, c)$

In Logic (informally)

- Basic facts: **axioms** (definitions)
- Derived facts: **theorems**

Independent axiom
an axiom that cannot be derived from the rest
$\longrightarrow$ Goal of mathematicians: find the minimal set of independent axioms

In AI

- Assertions: sentences added to a KB using TELL
- Queries or goals: sentences asked to KB using ASK

Prepare for Monday:

AIMA, Exercise 7.2

Takes(x, H): student takes history course
Takes(x, B): student takes biology course
Fails(x, H): student fails history course
Fails(x, B): student fails biology course
Score(x, H): the score obtained by student x in history course
Score(x, B): the score obtained by student x in biology course
Disliked(x,y): person x dislikes person y
Shaves (x,y): person x shaves person y
Fools(x,y,t): person x fools person y at time t
Student (x), Person(x), Man (x), Woman(x), Barber (x),
Vegetarian(x), Smart(x), Professor(x), Politician(x),

AI Limerick

If your thesis is utterly vacuous
Use first-order predicate calculus
With sufficient formality
The sheerest banality
Will be hailed by the critics: "Miraculous!"

Henry Kautz

In Canadian Artificial Intelligence, September 1986

(then: University of Rochester

then: head of AI at AT&T Labs-Research

and Program co-chair of AAAI-2000 Now: Associate Professor at

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