

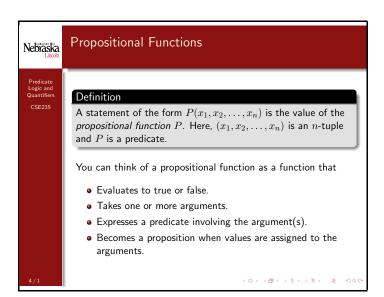
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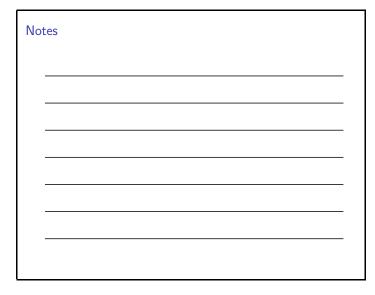
Nebraska Lincoln	Introduction
Predicate Logic and Quantifiers	Consider the following statements: $x > 3$, $x = y + 3$, $x + y = z$
CSE235	The truth value of these statements has no meaning without specifying the values of x,y,z .
	However, we can make propositions out of such statements.
	A <i>predicate</i> is a property that is affirmed or denied about the <i>subject</i> (in logic, we say "variable" or "argument") of a <i>statement</i> .
	" x subject is greater than 3" predicate
2/1	Terminology: affirmed = holds = is true; denied = does not hold = is not true. $ (3.66 \pm 0.000) = 0.000 = 0.0$

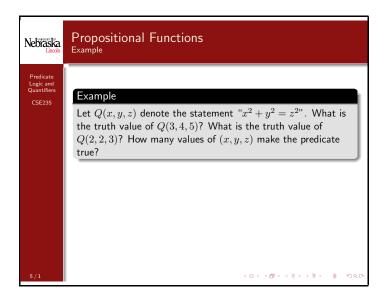
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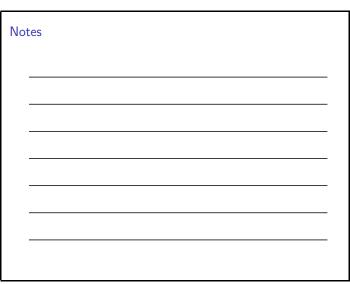
Nebraska Lincoln	Propositional Functions
Predicate Logic and Quantifiers CSE235	To write in predicate logic:
	" x subject is greater than 3" predicate
	We introduce a (functional) symbol for the predicate, and put the subject as an argument (to the functional symbol): $P(x)$
	Examples:
	 Father(x): unary predicate Brother(x,y): binary predicate Sum(x,y,z): ternary predicate P(x,y,z,t): n-ary predicate
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Propositional Functions Example

Example

Example

Example

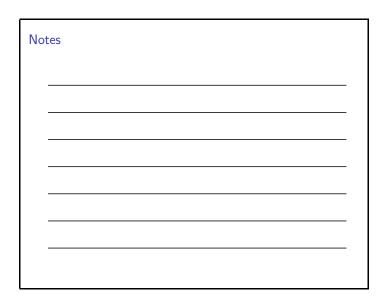
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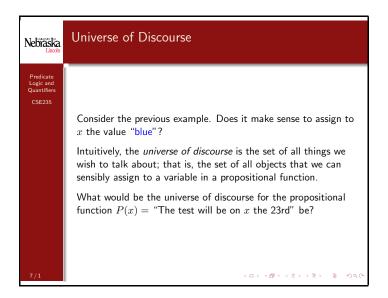
Let Q(x,y,z) denote the statement " $x^2+y^2=z^2$ ". What is the truth value of Q(2,2,3)? What is the truth value of Q(2,2,3)? How many values of (x,y,z) make the predicate true?

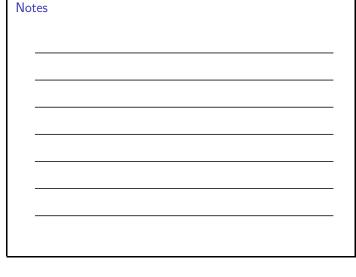
Since $3^2+4^2=25=5^2$, Q(3,4,5) is true.

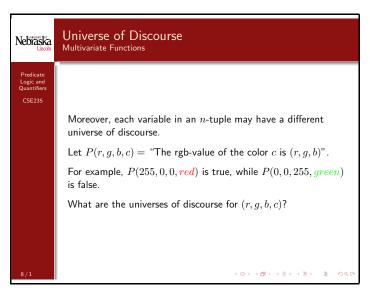
Since $2^2+2^2=8\neq 3^2=9$, Q(2,2,3) is false.

There are infinitely many values for (x,y,z) that make this propositional function true—how many right triangles are there?

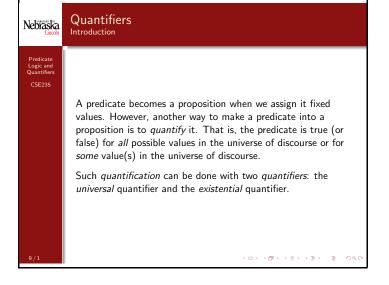




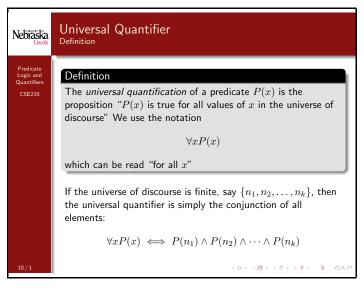


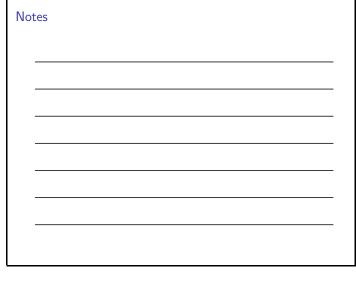


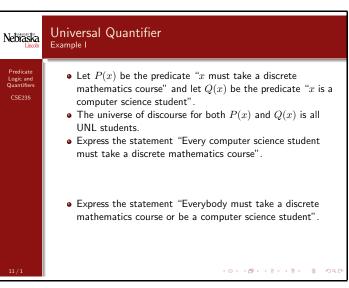
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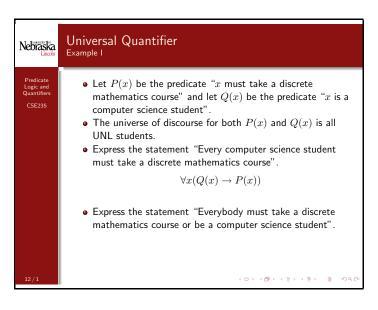
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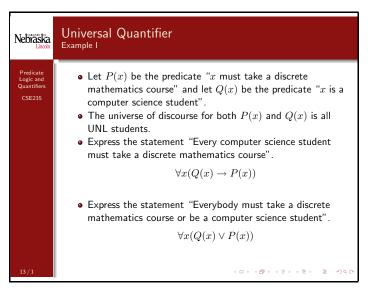


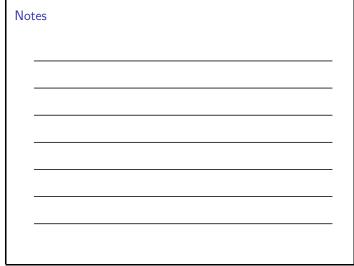


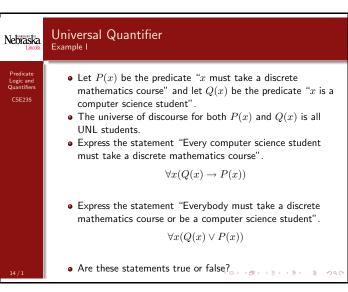
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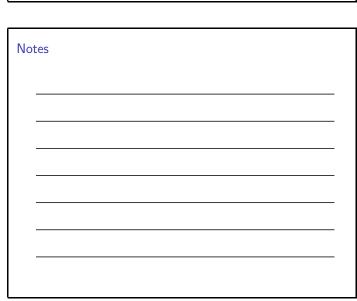


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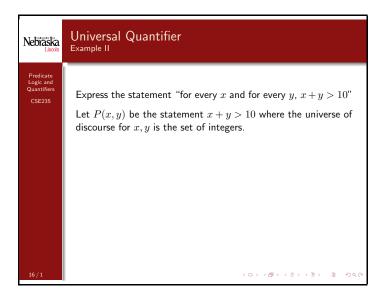


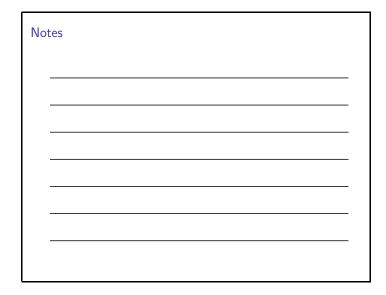




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Predicate Logic and Quantifiers CSE235	Express the statement "for every x and for every $y,x+y>10$ "
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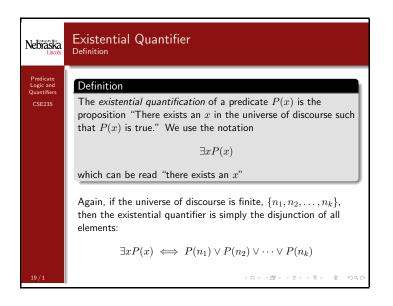


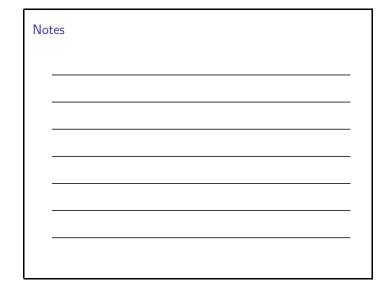
Nebraska Lincoln	Universal Quantifier Example II
Predicate Logic and Quantifiers CSE235	Express the statement "for every x and for every $y, x+y>10$ " Let $P(x,y)$ be the statement $x+y>10$ where the universe of discourse for x,y is the set of integers. Answer: $\forall x \forall y P(x,y)$
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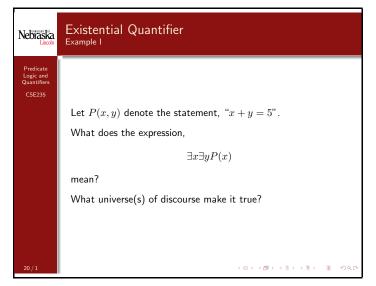
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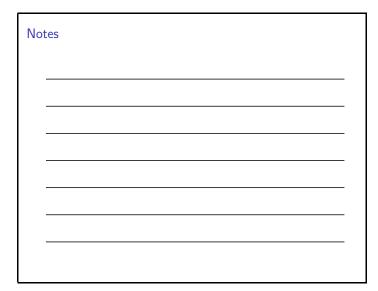
Nebraska Lincoln	Universal Quantifier Example II
Predicate Logic and Quantifiers CSE235	Express the statement "for every x and for every $y, x+y>10$ " Let $P(x,y)$ be the statement $x+y>10$ where the universe of discourse for x,y is the set of integers. Answer: $\forall x \forall y P(x,y)$
18 / 1	Note that we can also use the shorthand $\forall x,y P(x,y)$

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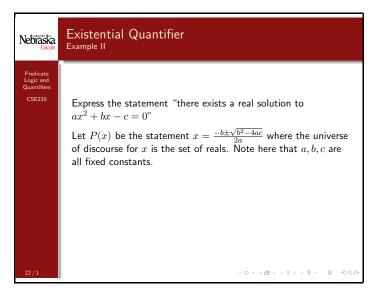


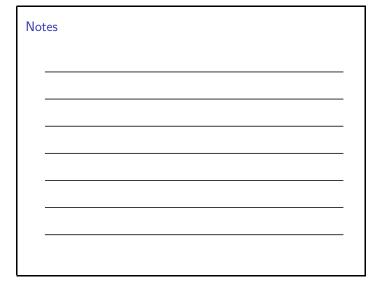




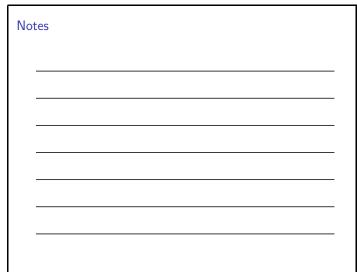
Nebraska Lincoln	Existential Quantifier Example II
Predicate Logic and Quantifiers CSE235	Express the statement "there exists a real solution to $ax^2+bx-c=0 \label{eq:constraint}$
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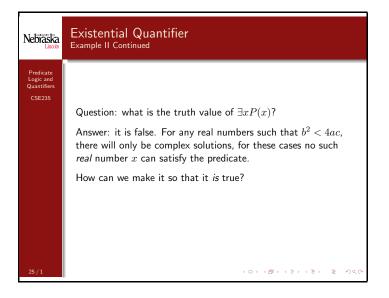


Nebraska Lincoln	Existential Quantifier Example II				
Predicate Logic and Quantifiers					
CSE235	Express the statement "there exists a real solution to $ax^2+bx-c=0 \label{eq:constraint}$				
	Let $P(x)$ be the statement $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ where the universe of discourse for x is the set of reals. Note here that a,b,c are all fixed constants.				
	The statement can thus be expressed as				
	$\exists x P(x)$				
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Nebraska Lincoln	Existential Quantifier Example II Continued
Predicate Logic and Quantifiers CSE235	Question: what is the truth value of $\exists x P(x)$?
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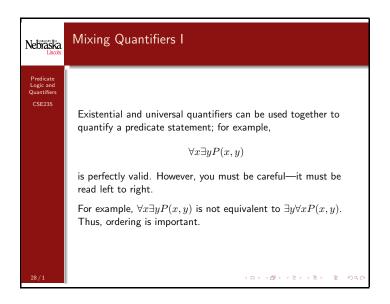
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Nebraska Lincoln	Existential Quantifier Example II Continued
Predicate Logic and Quantifiers CSE235	
	Question: what is the truth value of $\exists x P(x)$?
	Answer: it is false. For any real numbers such that $b^2 < 4ac$, there will only be complex solutions, for these cases no such real number x can satisfy the predicate.
	How can we make it so that it is true?
	Answer: change the universe of discourse to the complex numbers, $\mathbb{C}.$
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Nebraska Lincoln	Quantifiers Truth Values		
Predicate Logic and Quantifiers CSE235	In general, whe	n are quantified stateme	ents true/false?
	Statement	True When	False When
	$\forall x P(x)$	P(x) is true for every x .	There is an x for which $P(x)$ is false.
	$\exists x P(x)$	There is an x for which $P(x)$ is true.	P(x) is false for every x .
		Table: Truth Values of Q	uantifiers
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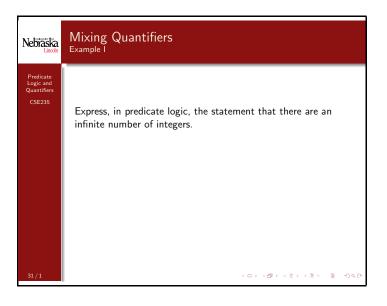
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Nebraska Lincoln	Mixing Quantifiers II
Predicate Logic and Quantifiers CSE235	For example: $ \forall x \exists y Loves(x,y) \text{: everybody loves somebody} $ $ \exists y \forall x Loves(x,y) \text{: There is someone loved by everyone} $
	Those expressions do not mean the same thing! Note that $\exists y \forall x P(x,y) \to \forall x \exists y P(x,y)$, but the converse does not hold However, you can commute $similar$ quantifiers; $\exists x \exists y P(x,y)$ is equivalent to $\exists y \exists x P(x,y)$ (which is why our shorthand was valid).
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Nebraska Lincoln	Mixing Quantifiers Truth Values								
Predicate Logic and	Statement	True When	False When						
Quantifiers	$\forall x \forall y P(x,y)$	P(x,y) is true for ev-	There is at least one						
CSE235		ery pair x, y .	pair, x,y for which $P(x,y)$ is false.						
	$\forall x \exists y P(x,y)$	For every x , there is a y for which $P(x,y)$ is	There is an x for which $P(x,y)$ is false						
		true.	for every y .						
	$\exists x \forall y P(x,y)$	There is an x for which $P(x,y)$ is true for every y .	For every x , there is a y for which $P(x,y)$ is false.						
	$\exists x \exists y P(x,y)$	There is at least one pair x,y for which $P(x,y)$ is true.	P(x,y) is false for every pair x,y .						
	Tab	le: Truth Values of 2-varia	te Quantifiers						
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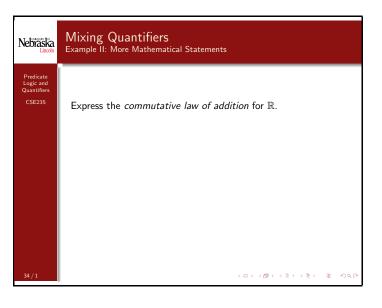
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Nebraska Lincoln	Mixing Quantifiers Example I
Predicate Logic and Quantifiers CSE235	Express, in predicate logic, the statement that there are an infinite number of integers. Let $P(x,y)$ be the statement that $x < y$. Let the universe of discourse be the integers, $\mathbb Z$.
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Nebraska Lincoln	Mixing Quantifiers Example I
Predicate Logic and Quantifiers CSE235	
	Express, in predicate logic, the statement that there are an infinite number of integers.
	Let $P(x,y)$ be the statement that $x < y$. Let the universe of discourse be the integers, \mathbb{Z} .
	Then the statement can be expressed by the following.
	$\forall x \exists y P(x,y)$
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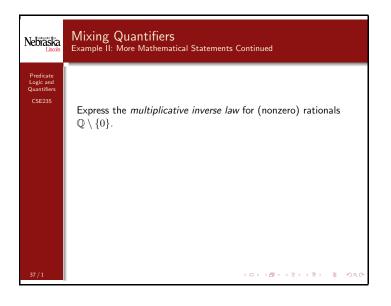
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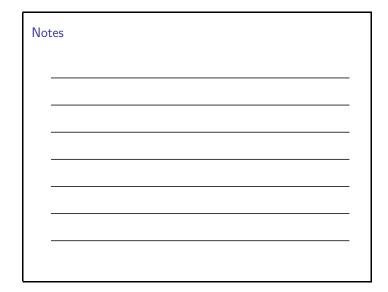
Nebraska Lincoln	Mixing Quantifiers Example II: More Mathematical Statements
Predicate Logic and Quantifiers CSE235	Europea the commutative law of addition for ID
	Express the <i>commutative law of addition</i> for \mathbb{R} . We want to express that for every pair of reals, x,y the
	following identity holds: $x+y=y+x$
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Nebraska Lincoln	Mixing Quantifiers Example II: More Mathematical Statements
Predicate Logic and Quantifiers	
CSE235	Express the <i>commutative law of addition</i> for \mathbb{R} .
	We want to express that for every pair of reals, x,y the following identity holds:
	x + y = y + x
	Then we have the following:
	$\forall x \forall y (x+y=y+x)$
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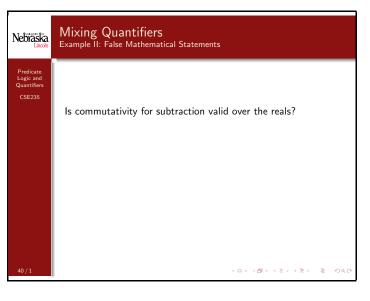


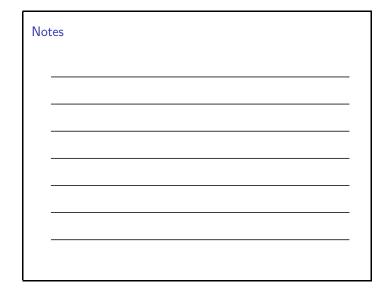
Nebraska Lincoln	Mixing Quantifiers Example II: More Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	Express the multiplicative inverse law for (nonzero) rationals $\mathbb{Q}\setminus\{0\}$. We want to express that for every real number x , there exists a real number y such that $xy=1$.
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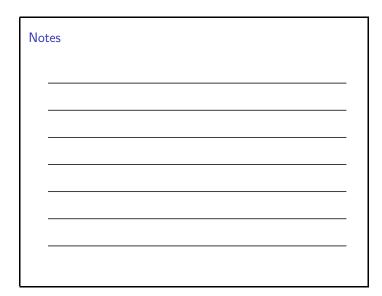
Nebraska Lincoln	Mixing Quantifiers Example II: More Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	Express the multiplicative inverse law for (nonzero) rationals $\mathbb{Q}\setminus\{0\}$. We want to express that for every real number x , there exists a real number y such that $xy=1$. Then we have the following:
39 / 1	$orall x \exists y (xy=1)$

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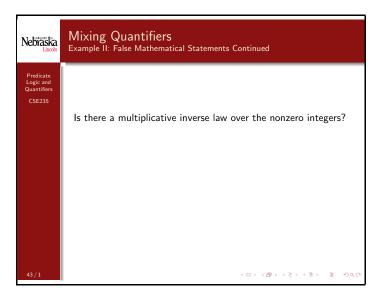


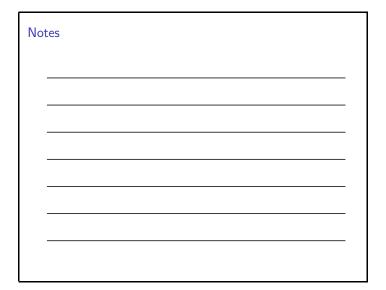
Nebraska Lincoln	Mixing Quantifiers Example II: False Mathematical Statements
Predicate Logic and Quantifiers CSE235	
	Is commutativity for subtraction valid over the reals?
	That is, for all pairs of real numbers x,y does the identity $x-y=y-x$ hold? Express this using quantifiers.
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Nebraska Lincoln	Mixing Quantifiers Example II: False Mathematical Statements
Predicate Logic and Quantifiers CSF235	
CSEESS	Is commutativity for subtraction valid over the reals?
	That is, for all pairs of real numbers x,y does the identity $x-y=y-x$ hold? Express this using quantifiers.
	The expression is
	$\forall x \forall y (x - y = y - x)$
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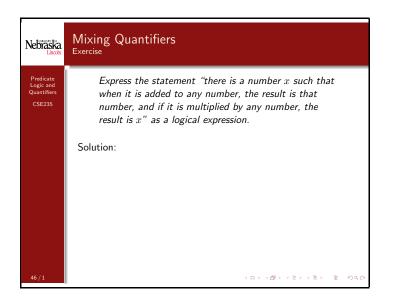


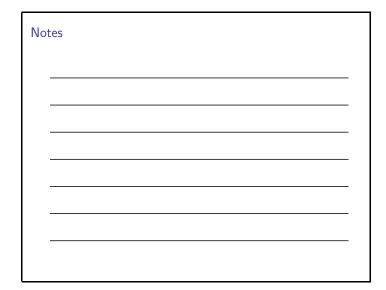
Nebraska Lincoln	Mixing Quantifiers Example II: False Mathematical Statements Continued
Predicate Logic and Quantifiers CSE235	
	Is there a multiplicative inverse law over the nonzero integers?
	That is, for every integer x does there exists a y such that $xy=1$?
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Nebraska Lincoln	Mixing Quantifiers Example II: False Mathematical Statements Continued					
Predicate Logic and Quantifiers CSE235						
	Is there a multiplicative inverse law over the nonzero integers?					
	That is, for every integer x does there exists a y such that $xy=1$?					
	This is false, since we can find a <i>counter example</i> . Take any integer, say 5 and multiply it with another integer, y . If the statement held, then $5=1/y$, but for any (nonzero) integer y , $ 1/y \leq 1$.					
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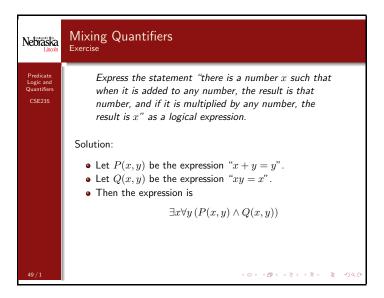


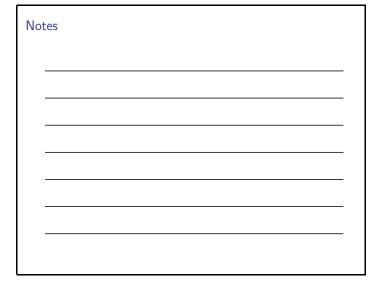
Nebraska Lincoln	Mixing Quantifiers Exercise
Predicate Logic and Quantifiers CSE235	Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x " as a logical expression.
	Solution:
	ullet Let $P(x,y)$ be the expression " $x+y=y$ ".
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Nebraska Lincoln	Mixing Quantifiers Exercise
Predicate Logic and Quantifiers CSE235	Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x " as a logical expression.
	Solution: • Let $P(x,y)$ be the expression " $x+y=y$ ". • Let $Q(x,y)$ be the expression " $xy=x$ ".
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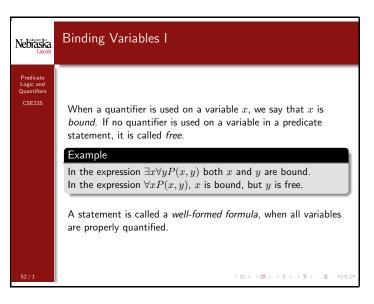


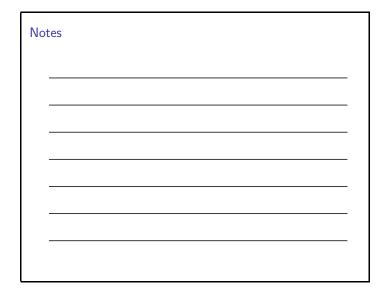
Nebrasia Exercise Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x" as a logical expression. Solution: Let P(x,y) be the expression "x+y=y". Let Q(x,y) be the expression "xy=x". Then the expression is $\exists x \forall y \, (P(x,y) \land Q(x,y))$ Over what universe(s) of discourse does this statement hold?

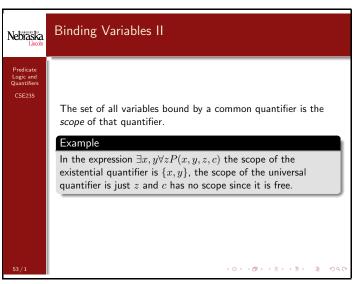
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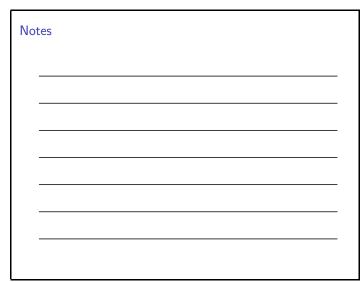
Nebraska Lincoln	Mixing Quantifiers Exercise
Predicate Logic and Quantifiers CSE235	Express the statement "there is a number x such that when it is added to any number, the result is that number, and if it is multiplied by any number, the result is x'' as a logical expression.
	Solution:
	• Let $P(x,y)$ be the expression " $x+y=y$ ". • Let $Q(x,y)$ be the expression " $xy=x$ ". • Then the expression is
	$\exists x \forall y \left(P(x,y) \land Q(x,y) \right)$
	 Over what universe(s) of discourse does this statement hold?
51 / 1	• This is the <i>additive identity law</i> and holds for $\mathbb{N}, \mathbb{Z}, \mathbb{R}, \mathbb{Q}$ but does not hold for \mathbb{Z}^+ .

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Nebraska Lincoln	Negation
Predicate Logic and Quantifiers CSE235	Just as we can use negation with propositions, we can use them with quantified expressions.
	Let $P(x)$ be a predicate. Then the following hold.
	$\neg \forall x P(x) \equiv \exists x \neg P(x)$
	$\neg \exists x P(x) \equiv \forall x \neg P(x)$
	This is essentially a quantified version of De Morgan's Law (in fact if the universe of discourse is finite, it is <i>exactly</i> De Morgan's law).
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Negation Truth Values Nebraska Predicate Logic and Quantifiers CSE235 Statement True When False When $\neg \exists x P(x) \equiv |$ For every x, P(x) is | There is an x for $\forall x \neg P(x)$ which P(x) is true. false. $\neg \forall x P(x) \equiv$ There is an x for P(x) is true for every $\exists x \neg P(x)$ which P(x) is false. Table: Truth Values of Negated Quantifiers ←□→ ←□→ ←□→ ←□→ □□ → □

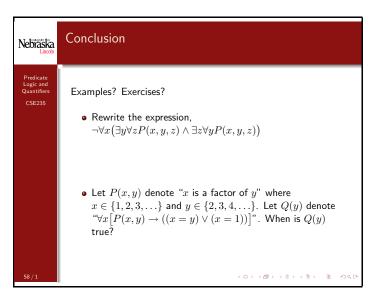
Prolog Prolog (Programming in Logic) is a programming language based on (a restricted form of) Predicate Calculus. It was developed by the logicians of the artificial intelligence community for symbolic reasoning.	
Quantifiers based on (a restricted form of) Predicate Calculus. It was developped by the logicians of the artificial intelligence	
 Prolog allows the user to express facts and rules Facts are proposational functions: student(juana), enrolled(juana,cse235), instructor(patel,cse235), etc. Rules are implications with conjunctions: teaches(X,Y): instructor(X,Z), enrolled(Y,Z) Prolog answers queries such as: ?enrolled(juana,cse478) ?enrolled(X,cse478) ?teaches(X,juana) by binding variables and doing theorem proving (i.e., applying inference rules) as we will see in Section 1.5. 	

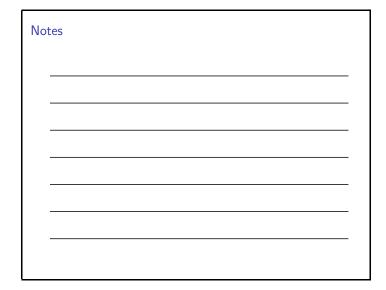
Nebraska Lincoln	English into Logic
Predicate Logic and Quantifiers CSE235	Logic is more precise than English. Transcribing English to Logic and vice versa can be tricky. When writing statements with quantifiers, usually the correct meaning is conveyed with the following combinations: • Use \forall with \Rightarrow Example: $\forall xLion(x) \Rightarrow Fierce(x)$ $\forall xLion(x) \land Fierce(x)$ means "everyone is a lion and everyone is fierce" • Use \exists with \land Example: $\exists xLion(x) \land Drinks(x, coffee)$: holds when you have at least one lion that drinks coffee $\exists xLion(x) \Rightarrow Drinks(x, coffee)$ holds when you have people even though no lion drinks coffee.
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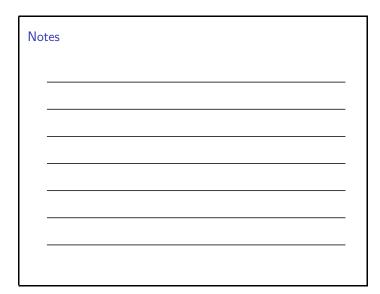
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Nebraska Lincoln	Conclusion
Predicate Logic and Quantifiers CSE235	Examples? Exercises? • Rewrite the expression, $\neg \forall x \big(\exists y \forall z P(x,y,z) \land \exists z \forall y P(x,y,z)\big)$ • Answer: Use the negated quantifiers and De Morgan's law. $\exists x \big(\forall y \exists z \neg P(x,y,z) \lor \forall z \exists y \neg P(x,y,z)\big)$ • Let $P(x,y)$ denote " x is a factor of y " where $x \in \{1,2,3,\ldots\}$ and $y \in \{2,3,4,\ldots\}$. Let $Q(y)$ denote " $\forall x \big[P(x,y) \rightarrow ((x=y) \lor (x=1))\big]$ ". When is $Q(y)$
59 / 1	true?



Nebraska Lincoln	Conclusion
Predicate Logic and Quantifiers CSE235	Examples? Exercises? • Rewrite the expression, $\neg \forall x (\exists y \forall z P(x,y,z) \land \exists z \forall y P(x,y,z))$
	• Answer: Use the negated quantifiers and De Morgan's law. $\exists x \big(\forall y \exists z \neg P(x,y,z) \lor \forall z \exists y \neg P(x,y,z) \big)$ • Let $P(x,y)$ denote " x is a factor of y " where $x \in \{1,2,3,\ldots\} \text{ and } y \in \{2,3,4,\ldots\}. \text{ Let } Q(y) \text{ denote } "\forall x \big[P(x,y) \to ((x=y) \lor (x=1)) \big]$ ". When is $Q(y)$ true? • Answer: Only when y is a prime number.
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Extra Question

Predicate Logic and Quantifiers CSE235

Some students wondered if

$$\forall x, y P(x,y) \equiv \forall x P(x,y) \land \forall y P(x,y)$$

This is certainly not true. In the left-hand side, both x and y are bound. In the right-hand side, x is bound in the first predicate, but y is free. In the second predicate, y is bound but x is free.

All variables that occur in a propositional function must be bound to turn it into a proposition.

Thus, the left-hand side is a proposition, but the right-hand side is not. How can they be equivalent?

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