

CSCE 310J: Data Structures & Algorithms



Logic Summary

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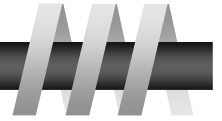
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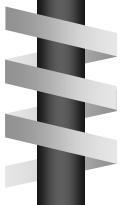
∞ Giving credit where credit is due:

- **Most of slides for this lecture are based on slides created by Dr. Ben Choi, Louisiana Technical University.**
- **I have modified them and added new slides**

Analysis Tool: Logic



- ∩ **Logic is a system for formalizing natural language statements so that we can reason more accurately.**
- ∩ **The simplest statements are called *atomic formulas*.**
- ∩ **More complex statements can be build up through the use of *logical connectives*: \wedge “and”, \vee “or”, \neg “not”, \Rightarrow “implies” $A \Rightarrow B$ “A implies B” “if A then B”**
- ∩ **$A \Rightarrow B$ is logically equivalent to $\neg A \vee B$**
- ∩ **DeMorgan’s laws:**
 - **$\neg (A \wedge B)$ is logically equivalent to $\neg A \vee \neg B$**
 - **$\neg (A \vee B)$ is logically equivalent to $\neg A \wedge \neg B$**



Quantifiers: all, some

∩ “for all x” $\forall x P(x)$ is true iff $P(x)$ is true for *all* x

- universal quantifier (universe of discourse)

∩ “there exist x” $\exists x P(x)$ is true iff $P(x)$ is true for *some* value of x

- existential quantifier

∩ $\forall x A(x)$ is logically equivalent to $\neg \exists x(\neg A(x))$

∩ $\exists x A(x)$ is logically equivalent to $\neg \forall x(\neg A(x))$

∩ $\forall x (A(x) \Rightarrow B(x))$

“For all x such that if A(x) holds then B(x) holds”

Prove by counterexample, Contraposition, Contradiction



∩ *Counterexample*

to prove $\forall x (A(x) \Rightarrow B(x))$ is false, we show *some* object x for which $A(x)$ is true and $B(x)$ is false.

- $\neg(\forall x (A(x) \Rightarrow B(x))) \Leftrightarrow \exists x (A(x) \wedge \neg B(x))$

∩ *Contraposition*

to prove $A \Rightarrow B$, we show $(\neg B) \Rightarrow (\neg A)$

∩ *Contradiction*

to prove $A \Rightarrow B$, we assume $\neg B$ and then prove B .

- $A \Rightarrow B \Leftrightarrow (A \wedge \neg B) \Rightarrow B$
- $A \Rightarrow B \Leftrightarrow (A \wedge \neg B)$ is false

∩ Assuming $(A \wedge \neg B)$ is true,
and discover a *contradiction* (such as $A \wedge \neg A$),
then conclude $(A \wedge \neg B)$ is false, and so $A \Rightarrow B$.

Prove by Contradiction Example

∩ Prove $[B \wedge (B \Rightarrow C)] \Rightarrow C$

- by contradiction

∩ **Proof:**

Assume $\neg C$

$$\neg C \wedge [B \wedge (B \Rightarrow C)]$$

$$\Rightarrow \neg C \wedge [B \wedge (\neg B \vee C)]$$

$$\Rightarrow \neg C \wedge [(B \wedge \neg B) \vee (B \wedge C)]$$

$$\Rightarrow \neg C \wedge [(B \wedge C)]$$

$$\Rightarrow \neg C \wedge C \wedge B$$

$$\Rightarrow \text{False, Contradiction}$$

$$\Rightarrow C$$

Rules of Inference

- ∩ **A rule of inference is a *general pattern* that allows us to draw some new conclusion from a set of given statements.**
 - If we know {...} then we can conclude {...}
- ∩ **If {**B and (B \Rightarrow C)**} then {**C**}**
 - modus ponens
- ∩ **If {**A \Rightarrow B and B \Rightarrow C**} then {**A \Rightarrow C**}**
 - syllogism
- ∩ **If {**B \Rightarrow C and \neg B \Rightarrow C**} then {**C**}**
 - rule of cases

Boolean Algebra



A set B and two binary operators $+$ “or, \vee ”, \cdot “and, \wedge ” creates a Boolean Algebra iff Huntington’s Postulates hold:

1. Closure:

- if $x, y \in B$ and $z = x + y$ then $z \in B$
- if $x, y \in B$ and $z = x \cdot y$ then $z \in B$

2. Identity and Zero elements:

- for $+$ designated by 0 : $x + 0 = x$
- for \cdot designated by 1 : $x \cdot 1 = x$

3. Commutative:

- $x + y = y + x$
- $x \cdot y = y \cdot x$

4. Distributive:

- $x \cdot (y + z) = (x \cdot y) + (x \cdot z)$
- $x + (y \cdot z) = (x + y) \cdot (x + z)$

5. Complement: for every element $x \in B$, there exists an element $x' \in B$ s.t.

- $x + x' = 1, x \cdot x' = 0$

6. There are at least two distinct elements in B

Truth Table and Implication Tautology (theorem)

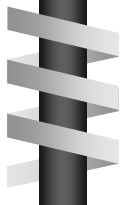


∴ Show $[B \wedge (B \Rightarrow C)] \Rightarrow C$ is a tautology:

•	B	C	$(B \Rightarrow C)$	$[B \wedge (B \Rightarrow C)]$	$[B \wedge (B \Rightarrow C)] \Rightarrow C$
•	0	0	1	0	1
•	0	1	1	0	1
•	1	0	0	0	1
•	1	1	1	1	1

∴ For every assignment for B and C,

- the statement is True



Prove by Rules of Inference

∩ **Prove** $[B \wedge (B \Rightarrow C)] \Rightarrow C$

• **Proof:**

$$[B \wedge (B \Rightarrow C)] \Rightarrow C$$

$$\Rightarrow \neg[B \wedge (B \Rightarrow C)] \vee C$$

$$\Rightarrow \neg[B \wedge (\neg B \vee C)] \vee C$$

$$\Rightarrow \neg[(B \wedge \neg B) \vee (B \wedge C)] \vee C$$

$$\Rightarrow \neg[(B \wedge C)] \vee C$$

$$\Rightarrow \neg B \vee \neg C \vee C$$

$$\Rightarrow \text{True (tautology)}$$

∩ **Direct Proof:**

$$[B \wedge (B \Rightarrow C)] \Rightarrow [B \wedge C] \Rightarrow C$$