

# CSCE 451/851

## Operating Systems Principles

### Page Replacement Algorithms

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#### Virtual Memory Management

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##### Fundamental issues

- ◆ Placement strategy
- ◆ Replacement strategies
- ◆ Load control strategies

Example: *Demand paging*

- » No placement strategy required *per se*
- » Load control: load pages only when faults occur
- » Replacement: ...

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## Page Replacement Algorithms

### Concept

- ◆ Typically  $\sum_i |VAS_i| \gg \text{Physical Memory}$
- ◆ With demand paging, physical memory fills quickly
- ◆ So when a process faults & memory is full, some page must be swapped out  
(So handling a page fault now requires 2 disk accesses not 1!)
- ◆ Which page should be replaced?
  - » *Local replacement* — replace a page of the faulting process
  - » *Global replacement* — possibly replace the page of another process

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## Page Replacement Algorithms

### Evaluation methodology

- ◆ Record a *trace* of pages accessed by a process
  - » Example: (Virtual) address trace...  
(3,0), (1,9), (4,1), (2,1), (5,3), (2,0), (1,9), (2,4), (3,1), (4,8)
  - » ... generates page trace  
3, 1, 4, 2, 5, 2, 1, 2, 3, 4 (represented as *c, a, d, b, e, b, a, b, c, d*)
- ◆ Simulate the behavior of a page replacement algorithm on the trace & record the number of page faults generated
  - » *fewer faults == better performance*

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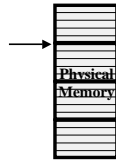
## Local Page Replacement FIFO replacement

- ◆ Simple to implement
  - » A single pointer suffices

- ◆ Performance with 4 page frames:

Page Trace

| Time        | 0        | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|-------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Requests    |          | <i>c</i> | <i>a</i> | <i>d</i> | <i>b</i> | <i>e</i> | <i>b</i> | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> |
| Page Frames |          | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>d</i> |
| 0           | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>d</i> |
| 1           | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> |
| 2           | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>b</i> | <i>b</i> | <i>b</i> |
| 3           | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>c</i> | <i>c</i> |
| Faults      |          |          |          |          |          | •        |          | •        | •        | •        | •        |



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## Optimal Page Replacement Clairvoyant replacement

- ◆ Replace the page that won't be needed for the longest time in the future

| Time                  | 0        | 1        | 2        | 3        | 4             | 5        | 6        | 7        | 8        | 9             | 10       |
|-----------------------|----------|----------|----------|----------|---------------|----------|----------|----------|----------|---------------|----------|
| Requests              |          | <i>c</i> | <i>a</i> | <i>d</i> | <i>b</i>      | <i>e</i> | <i>b</i> | <i>a</i> | <i>b</i> | <i>c</i>      | <i>d</i> |
| Page Frames           |          | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i>      | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i>      | <i>d</i> |
| 0                     | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i>      | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i>      | <i>d</i> |
| 1                     | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i>      | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i>      | <i>b</i> |
| 2                     | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i>      | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i>      | <i>c</i> |
| 3                     | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i>      | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i>      | <i>e</i> |
| Faults                |          |          |          |          |               |          | •        |          |          |               | •        |
| Time page needed next |          |          |          |          | <i>a</i> = 7  |          |          |          |          | <i>a</i> = 15 |          |
|                       |          |          |          |          | <i>b</i> = 6  |          |          |          |          | <i>b</i> = 11 |          |
|                       |          |          |          |          | <i>c</i> = 9  |          |          |          |          | <i>c</i> = 13 |          |
|                       |          |          |          |          | <i>d</i> = 10 |          |          |          |          | <i>d</i> = 14 |          |

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## Least Recently Used Replacement “Back to the future”

- ◆ Replace the page that hasn't been referenced for the longest time
  - » Use the recent past as a predictor of the future

| Time                | 0 | 1 | 2 | 3 | 4     | 5 | 6 | 7     | 8     | 9 | 10 |
|---------------------|---|---|---|---|-------|---|---|-------|-------|---|----|
| Requests            |   | c | a | d | b     | e | b | a     | b     | c | d  |
| Page Frames         |   |   |   |   |       |   |   |       |       |   |    |
| 0                   | a | a | a | a | a     | a | a | a     | a     | a | a  |
| 1                   | b | b | b | b | b     | b | b | b     | b     | b | b  |
| 2                   | c | c | c | c | c     | e | e | e     | e     | e | d  |
| 3                   | d | d | d | d | d     | d | d | d     | d     | e | c  |
| Faults              |   |   |   |   |       | • |   |       |       | • | •  |
| Time page last used |   |   |   |   | a = 2 |   |   | a = 7 | a = 7 |   |    |
|                     |   |   |   |   | b = 4 |   |   | b = 8 | b = 8 |   |    |
|                     |   |   |   |   | c = 1 |   |   | e = 5 | e = 5 |   |    |
|                     |   |   |   |   | d = 3 |   |   | d = 3 | c = 9 |   |    |

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## Least Recently Used Replacement Implementation

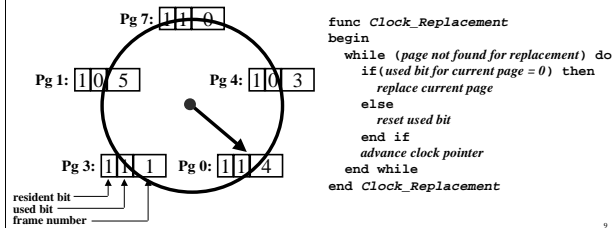
- ◆ Maintain a “stack” of recently used pages

| Time           | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------|---|---|---|---|---|---|---|---|---|---|----|
| Requests       |   | c | a | d | b | e | b | a | b | c | d  |
| Page Frames    |   |   |   |   |   |   |   |   |   |   |    |
| 0              | a | a | a | a | a | a | a | a | a | a | a  |
| 1              | b | b | b | b | b | b | b | b | b | b | b  |
| 2              | c | c | c | c | c | e | e | e | e | e | d  |
| 3              | d | d | d | d | d | d | d | d | d | e | c  |
| Faults         |   |   |   |   |   |   | • |   |   | • | •  |
| LRU Page Stack |   |   |   |   |   |   |   |   |   |   |    |
|                |   |   |   | c | a | a | d | d | e | a | a  |
|                |   |   | c | a | d | d | e | e | a | b | b  |
|                | c | a | d | b | e | b | a | b | c | d |    |

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## Approximate *LRU* Replacement The *Clock* algorithm

- ◆ Maintain a circular list of pages resident in memory
  - » Use a *clock* (or *used*) bit to track how often a page is accessed — bit set whenever a page is referenced
- ◆ Clock sweeps over pages looking for one with used bit = 0
  - » Replaces pages that haven't been referenced for one complete revolution of the clock



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## Clock Page Replacement Example

| Time        | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-------------|---|---|---|---|---|---|---|---|---|---|----|
| Requests    |   | c | a | d | b | e | b | a | b | c | d  |
| Page Frames | 0 | a | a | a | a | e | e | e | e | e | d  |
| 1           | b | b | b | b | b | b | b | b | b | b | b  |
| 2           | c | c | c | c | c | c | a | a | a | a | a  |
| 3           | d | d | d | d | d | d | d | d | d | c | c  |

Faults

Page table entries  
for resident pages:

|   |   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | a | 1 | e | 1 | e | 1 | e | 1 | e | 1 | d |
| 1 | b | 0 | b | 1 | b | 1 | b | 1 | b | 1 | b |
| 1 | c | 0 | c | 0 | c | 1 | a | 1 | a | 1 | a |
| 1 | d | 0 | d | 0 | d | 0 | d | 0 | d | 1 | c |

The LRU page

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## Performance of Page Replacement Algorithms

- ◆ Least recently used
  - » Ages pages based on when they were last used
- ◆ FIFO
  - » Ages pages based on when they're brought into memory

*The principle of locality*

- ◆ 90% of the execution of a program is sequential
- ◆ Most interactive constructs consist of a relatively small number of instructions
- ◆ When processing large data structures, the dominant cost is sequential processing on individual structure elements

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## Belady's Anomaly More memory ≠ Better performance!

### FIFO Replacement

| Time           | 0        | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       | 11       | 12       |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Requests       | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>a</i> | <i>b</i> | <i>e</i> | <i>a</i> | <i>b</i> | <i>c</i> | <i>d</i> | <i>e</i> |          |
| Page<br>Frames | 0        | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> |
|                | 1        | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>c</i> | <i>c</i> | <i>c</i> |
|                | 2        | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>d</i> | <i>d</i> |
| Faults         |          |          |          |          |          | •        | •        | •        | •        |          |          | •        | •        |
| Page<br>Frames | 0        | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>e</i> | <i>d</i> | <i>d</i> |
|                | 1        | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>a</i> | <i>e</i> |
|                | 2        | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>c</i> | <i>b</i> | <i>b</i> | <i>b</i> | <i>b</i> |
|                | 3        | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>d</i> | <i>c</i> | <i>c</i> | <i>c</i> |
| Faults         |          | •        |          |          |          |          | •        |          |          | •        | •        | •        | •        |

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## Explicitly Using Locality

### The working set model of page replacement

- ◆ Assume recently referenced pages are likely to be referenced again soon...
- ◆ ... and only keep those pages recently referenced in memory (called *the working set*)
  - » pages may be removed even when no page fault occurs
  - » number of frames allocated to a process will vary over time
- ◆ A process is allowed to execute only if its working set fits into memory
  - » implicit load control

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## Working Set Page Replacement Implementation

- ◆ Keep track of the last  $\tau$  references
  - » The pages referenced during the last  $\tau$  memory accesses are the working set
  - »  $\tau$  is called the *window size*
- ◆ Example: working set computation,  $\tau = 4$  references:

| Time          | 0           | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|---------------|-------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Requests      |             | <i>c</i> | <i>c</i> | <i>d</i> | <i>b</i> | <i>c</i> | <i>e</i> | <i>c</i> | <i>e</i> | <i>a</i> | <i>d</i> |
| Page <i>a</i> | $\tau_{-0}$ | *        | *        | *        | -        | -        | -        | -        | -        | *        | *        |
| Page <i>b</i> | -           | -        | -        | -        | *        | *        | *        | *        | -        | -        | -        |
| Page <i>c</i> | -           | *        | *        | *        | *        | *        | *        | *        | *        | *        | *        |
| Page <i>d</i> | $\tau_{-1}$ | *        | *        | *        | *        | *        | *        | *        | *        | *        | *        |
| Page <i>e</i> | $\tau_{-2}$ | *        | -        | -        | -        | -        | *        | *        | *        | *        | *        |
| Faults        |             | *        |          |          | *        |          | *        |          |          | *        | *        |

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## Optimal Page Replacement

### For processes with a variable number of frames

- ◆ *VMIN* — Replace a page that is not referenced in the *next*  $\tau$  accesses

- ◆ Example:  $\tau = 4$

| Time               | 0             | 1         | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|--------------------|---------------|-----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Requests           |               | <i>c</i>  | <i>c</i> | <i>d</i> | <i>b</i> | <i>c</i> | <i>e</i> | <i>c</i> | <i>e</i> | <i>a</i> | <i>d</i> |
| Pages<br>in Memory | Page <i>a</i> | $t_{a=0}$ | -        | -        | -        | -        | -        | -        | -        | •        | -        |
|                    | Page <i>b</i> | -         | -        | -        | -        | •        | -        | -        | -        | -        | -        |
|                    | Page <i>c</i> | -         | •        | •        | •        | •        | •        | •        | -        | -        | -        |
|                    | Page <i>d</i> | •         | •        | •        | •        | -        | -        | -        | -        | -        | •        |
|                    | Page <i>e</i> | -         | -        | -        | -        | -        | -        | •        | •        | -        | -        |
| Faults             |               | •         |          |          | •        |          | •        |          |          | •        | •        |

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## Page-Fault-Frequency Page Replacement

### An alternate working set computation

- ◆ Explicitly attempt to minimize page faults
  - » When page fault frequency is high — *increase working set*
  - » When page fault frequency is low — *decrease working set*
- ◆ Algorithm:
  - » Keep track of the rate at which faults occur
    - ❖ Record the time,  $t_{last}$ , of the last page fault
  - » If the time between page faults is “large” then reduce working set
    - ❖ If  $t_{current} - t_{last} > \tau$ , then remove from memory all pages not referenced in  $[t_{last}, t_{current}]$
  - » If the time between page faults is “small” then increase working set
    - ❖ If  $t_{current} - t_{last} \leq \tau$ , then add faulting page to the working set

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## Page-Fault-Frequency Page Replacement Example, window size = 2

- ◆ If  $t_{current} - t_{last} > 2$ , remove pages not referenced in  $[t_{last}, t_{current}]$  from the working set
- ◆ If  $t_{current} - t_{last} \leq 2$ , add faulting page to the working set

| Time                  | 0                    | 1        | 2        | 3        | 4        | 5        | 6        | 7        | 8        | 9        | 10       |
|-----------------------|----------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| Requests              |                      | <i>c</i> | <i>c</i> | <i>d</i> | <i>b</i> | <i>c</i> | <i>e</i> | <i>c</i> | <i>e</i> | <i>a</i> | <i>d</i> |
| Pages<br>in<br>Memory | Page <i>a</i>        | •        | •        | •        | •        | -        | -        | -        | -        | •        | •        |
|                       | Page <i>b</i>        | -        | -        | -        | -        | •        | •        | •        | •        | -        | -        |
|                       | Page <i>c</i>        | -        | •        | •        | •        | •        | •        | •        | •        | •        | •        |
|                       | Page <i>d</i>        | •        | •        | •        | •        | •        | •        | •        | •        | -        | •        |
|                       | Page <i>e</i>        | •        | •        | •        | •        | -        | -        | •        | •        | •        | •        |
| <hr/>                 |                      |          |          |          |          |          |          |          |          |          |          |
|                       | Faults               | •        |          |          | •        |          | •        |          |          | •        | •        |
|                       | $t_{cur} - t_{last}$ |          |          |          | 3        |          | 2        |          |          | 3        | 1        |

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