Chapter 7

Conclusion

In this thesis, we investigated the structural complexity of some computational group-theoretic problems. The main results shown in this thesis provide more insights into the complexity of these problems.

In Chapter 4 of the thesis, we investigated the complexity of three basic computational group-theoretic problems; Membership Testing, Order Verification and Isomorphism Testing. All these problems are computationally difficult; there is no polynomial time algorithm for any of these problems over black-box groups. But, it is also not known whether they are hard for NP. In Chapter 4, we showed that these problems over solvable black-box groups are in the counting class SPP. This upper bound of SPP implies that these problems will not provide additional power as oracle to counting classes like PP, $C_{=}P$ and $\mathrm{Mod}_k P$. This is an indication in support of the belief that these problems are unlikely to be hard for NP. Another important aspect of this result is that these problems provide more examples of natural problems in SPP which are not known to be in P.

There is one major open question that arises from this investigation. Extended the above upper bound to problems defined over general black-box group. We believe that the problems we considered here are low for PP even over general black-box groups. The methods we used for solvable groups will not extend for the general case. We feel that new methods based on classification theorems about finite simple groups may be required for the proof of this. A more general question is to show membership of other natural problems in the complexity classes like SPP or LWPP.

In Chapters 5 and 6 of the thesis, our focus was on the complexity of learning some algebraic representation classes. Since Angluin's model of exact learning is not enough for a fine classification of their complexity, we proposed a new exact learning model called the teaching assistant model. For classifying the complexity of learning in this model, we introduced the notion of teaching assistant classes. Using these notions we were able to show that complexity of learning permutation groups and linear spaces over finite fields are quite different from that of learning 3-CNF. In Chapter 6 we considered separating teaching assistant classes using subclasses of the representation class of permutation groups.

We mention some open problems. Firstly, it will be interesting to see whether there are representation classes that can be learned with an LWPP-assistant but not with an SPP-assistant. Is it true that class SYM is polynomial-time learnable with an SPP-assistant? As a first step towards answering this, it will be interesting if one can show that the subclass of SYM consisting only of abelian groups as concepts, can be learned using an SPP-assistant.

The teaching assistant model for learning is as yet of only theoretical interest. As the next step, it will be interesting to develop more learning algorithms in this model for various other representation classes (not necessarily algebraic), and explore further the relationship between the teaching assistant model and other well-known models of learning.